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History of Mathematics: Models and Visualization in the Mathematical and Physical Sciences

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Abstract. This workshop brought together historians of mathematics and science as well as mathematicians to explore important historical developments connected with models and visual elements in the mathematical and physical sciences. It addressed the larger question of what has been meant by a model, a notion that has seldom been subjected to careful historical study. Most of the talks dealt with case studies from the period 1800 to 1950 that covered a number of analytical, geometrical, mechanical, astronomical, and physical phenomena. The workshop also considered the role of visual thinking as a component of mathematical creativity and understanding.

Mathematics Subject Classification (2010): 01A55, 01A60.

Introduction by the Organisers

The idea for this workshop came up in discussions at the history of mathematics workshop held at MFO during the week of March 3–9, 2013. Its aim was to bring a wide range of experts together in order to explore important historical developments connected with models and visual elements in the mathematical and physical sciences. Speakers focused on a number of case studies that dealt with visualizing geometrical, mechanical, astronomical, and physical phenomena during the period from roughly 1800 to 1950. Several talks discussed how visual models have functioned within purely mathematical disciplines. But just as many dealt with cases in bordering fields that employ mathematical theories and methods to study various physical phenomena.

A number of talks dealt with model-making in geometry during the latter half of the nineteenth century. Source materials describing the artefacts from this time, many on prominent display at the MFO, are quite plentiful. One can find much information about such string and plaster models from the catalogues of companies that produced them as well as the exhibition catalogues produced when they were put on display (South Kensington 1876, Munich 1893, etc.). More challenging for historians, however, is to understand the motivations behind this model-making activity. In most cases, the geometers who promoted it were teaching at the higher technical schools rather than at universities. Several were lesser known amateurs, whose work has been forgotten once the commercialization of geometrical models led to the proliferation of canonical artefacts.

Many speakers took note of the fact that the explicit use of the term model and/or modelling was not part of the original vocabulary of the actors themselves. Thus, the history of non-Euclidean geometry took an important turn with the work of Beltrami, Klein, and Poincaré. Yet none of these figure referred to "models" that they invented and which aimed to show the validity of the theories of Lobachevsky and Bolyai. Clearly, that terminology was taken up soon afterward, but not in their original publications. Likewise, in cosmology, the famous "models of Einstein and de Sitter" were originally referred to as "worlds". It seems likely that the term cosmological models did not become current until 1933, when H. P. Robertson used it in a widely read review article. These and other instances suggest that much of the retrospective literature has projected the terminology of mathematical modelling into earlier work, thereby distorting our view of its intentionality.

Philosophers of science have long been interested in the role of models in theory formation, whereas historians of mathematics have seldom paid close attention to the ways in which theoretical concerns are often entangled with concrete modelling activity. This workshop thus provided a welcome opportunity to explore the relationship between different representations of a phenomenon and their role in explanation. The year 1950 marks a natural boundary line for historical studies, since after then modern electronic computers opened vast new possibilities for mathematical modelling and visualization in the mathematical and physical sciences. In recent decades computer graphics have revolutionized the once largely static realm of visualizable mathematics. Models and simulations of complex phenomena have become so commonplace that one easily recognizes how radically different things were before the onset of the IT era. By looking at particular historical contexts and special cases, the workshop offered a clear sense of how models and visual thinking developed and reinforced one another. The diverse topics reflected in the abstracts below provide at least a provisional picture of how models and visual thinking shaped important historical developments.

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Mathematical Milky Way Models from Kelvin and Kapteyn to Poincaré, Jeans and Einstein

Scott A. Walter

Following William Thomson's calculation in 1901 of the Milky Way radius [9] and J. C. Kapteyn's announcement [5] at the Congress of Science and Arts during the World's Fair in Saint Louis of his discovery of two star-streams (1904), Henri Poincaré realized the interest of kinetic gas theory for modeling astronomical and cosmological phenomena. Soon others followed, including A. S. Eddington and Karl Schwarzschild, who proposed dualist and unitary models, respectively, of the observed stellar velocities. Eddington [1] affirmed Kapteyn's two-stream hypothesis on the basis of his analysis of the Groombridge stars, and claimed the streams were characterized by Maxwellian distributions with different constants. Shortly thereafter, Schwarzschild [8], on the basis of a different dataset, affirmed that there were not two star-streams but rather an ellipsoidal velocity distribution. The two models were judged at first to represent the data equally well, and further efforts were called for to determine which was best.

What Eddington and Schwarzschild provided in 1906–1907 were mathematical representations of empirical data. Neither Eddington nor Schwarzschild took up Poincaré's suggestion that the Milky Way was undergoing a rotation [6], at least not explicitly. Poincaré developed this bold conjecture in his Sorbonne lectures of 1910–1911 [7], the publication of which constituted the first theoretical treatise on cosmology. Notably, in his treatise Poincaré derived the virial for the case of a gaseous mass with Newtonian attraction, and took up the mixing problem. Like Poincaré, James Jeans challenged belief in the stationary state of the universe, based on his calculation of the angle of deflection of colliding stars [4]. A "stargas" (Sterngas) model of globular nebulæ was investigated by Einstein in 1921 using Poincaré's virial, presumably as a way to fix the value of the cosmological constant he had introduced in 1917 to the field equations of general relativity [2], and to obtain thereby an estimate of the size of the universe [3].

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Mach's Principle and Relativistic Cosmology, 1917–1924

David E. Rowe

This period marks the beginning of relativistic cosmology, which has normally been discussed in terms of two competing models: the "cylinder universe" of Einstein and the matter-free world of de Sitter. The term cosmological model only became common, however, after around 1933 when it was used in a well-known review paper written by H. P. Robertson. Einstein and de Sitter were concerned with finding static solutions to the field equations with "cosmological constant." In Einstein's case his universe aimed to implement what he called "Mach's Principle," a notion de Sitter rejected as pure speculation. The latter's matter–free universe flew in the face of Einstein's claim that the matter–field alone induced inertia, sparking a famous debate. Hermann Weyl and Felix Klein soon entered into this controversery, though in quite different ways. The period ends with Weyl's amusing dialogue, published in *Die Naturwissenschaften* in 1924, in which the debate is re-enacted as a theological discussion over the dogma of Mach's Principle as a condition for membership in the "church of relativity."

Einstein and de Sitter had already discussed the implications of general relativity for cosmology in 1916 when Einstein visited with him in Leyden. Einstein's first attempt to introduce a realivistic cosmology was based on a flat global space-time in which he let the gravitational potential become infinite at spatial infinity. Arguing against this, de Sitter noted that this assumption could not be made independent of the choice of coordinates, a point Einstein conceded in early 1917. It was then that he unveiled his famous "cylinder universe", a space-time geometry