Poincaré-Week in Göttingen, in light of the Hilbert-Poincaré correspondence of 1908–1909

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Forthcoming in Maria Teresa Borgato & Irène Passeron, eds., Scientific Correspondences, Springer

Introduction

At the turn of the twentieth century, two mathematicians, David Hilbert and Henri Poincaré were at the top of their field. As an indicator of their standing, recall that the first two Bolyai Prizes were awarded in sequence to Poincaré and Hilbert in 1905 and 1910, the latter on Poincaré's recommendation (Poincaré 1911). If Hilbert and Poincaré were both bright stars by any measure, their ideas about the nature of mathematics, and in particular, about the relation between mathematics and the phenomenal world, were quite dissimilar. Hilbert's pursuit of a formalist program, launched on the heels of the success of his axiomatization of Euclidean geometry (Hilbert 1899), could find no place in Poincaré's conventionalist worldview. On this basis, Mehrtens (1990) considered Hilbert and Poincaré to be polar opposites, as far as mathematical modernism is concerned. A more detailed examination of their respective contributions, however, reveals that both mathematicians contributed significantly to what Gray (2008) calls the "modernist transformation of mathematics".

Historically there are few instances in which Hilbert and Poincaré came into close personal contact. Similarly, there are few instances where their research interests overlapped, in contrast to the case presented by Hilbert's senior colleague in Göttingen, Felix Klein.[1] In the early 1880s, Klein and Poincaré engaged in a race of sorts to map out the new domain of automorphic functions (Gray 2000). The surviving correspondence between Klein

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[1]On Klein, Hilbert, and mathematics in Göttingen, see Rowe (1992).

and Poincaré is significant, counting twenty-six letters over a span of fifteen months ending in September, 1882. The surviving correspondence between Hilbert and Poincaré is less voluminous, with a total of ten letters, seven of which concern the arrangements for Poincaré's delivery of the first Wolfskehl lectures in April, 1909. This private exchange, which has been published on the website Henri Poincaré Papers, and is transcribed in the final section, offers insight into the Hilbert-Poincaré relationship, and into Poincaré's approach to questions of theoretical physics.

1 Hilbert's invitation

The summer of 1908 was a dark time for David Hilbert, as he was overcome by depression, and required several months of rest in a sanitorium in the Harz mountains. The cure was effective enough for him to resume his lectures in the fall, and to work out the first proof of Waring's Theorem (Reid 1986, 112). In the fall of 1908, his friend and colleague Hermann Minkowski was working on a theory of electrons, compatible with his successful spacetime theory of the electrodynamics of moving media. Minkowski's four-dimensional spacetime was inspired in part by Poincaré's Lorentz-covariant theory of gravitation, and yet there were significant differences in their conceptualizations of fundamental elements of their theories, having to do with the notion of time. And while Minkowski and Poincaré should have had an occasion to discuss the theory of relativity with Lorentz during the ICM in Rome in the spring (Walter 1999), Poincaré fell ill during the meeting, and was unable to deliver his plenary lecture.

It may be imagined that both Minkowski and Hilbert viewed the Rome ICM as a missed occasion. Whatever the case may be, Hilbert wrote to Poincaré in the fall of 1908, inviting him to deliver a series of lectures under the auspices of the Wolfskehl-Stiftung, which provided a substantial honorarium of 2500 Marks (Hilbert 1910). Poincaré accepted the invitation, but neither Hilbert's letter of invitation nor Poincaré's response has been located. Hilbert thanked Poincaré for accepting his invitation, and suggested that he begin his lecture series in late February or late April (§ 4.1). Poincaré replied that he was about to be inducted into the Académie Française, but he did not know precisely when this would take place (§ 4.2). On November 19, Hilbert interpreted Poincaré's response to mean that he preferred to lecture in late April, and expressed his "great interest" in receiving his lecture program.

2 Poincaré's lecture program

Up to this point, the Hilbert-Poincaré exchange was perfunctory, but now the terms of the encounter between Poincaré and the Göttingen mathematicians were to be fixed. Sometime after the 19th of November, Poincaré wrote to Hilbert to advise him of his lecture program. This program included two topics: applications of Fredholm's method, and the reduction of Abelian integrals. Poincaré expressed his wish to retain the power to modify his program, "if need be" (§ 4.4).

Hilbert may have taken awhile to respond to Poincaré's program. On 12 January, 1909, his good friend Minkowski, who was "a thousand times more a brother" to him, died suddenly from a ruptured appendix (§ 4.5). This "blow out of left field" *(Schlag aus dem heitersten Himmel)* effectively mooted any ideas Hilbert might have had about getting Minkowski and Poincaré together.

Undoubtedly, others in and about Göttingen were still interested in discussing Poincaré's electron theory with Poincaré, among other topics. The only problem was that Poincaré had not offered to lecture on this topic, or others in theoretical physics. Ostensibly in the interest of enticing physicists, astronomers, and logicians to attend what Hilbert dubbed "Poincaré-Week", Hilbert asked Poincaré to add two topics to those he had already proposed. One might be on theoretical physics or astronomy, the other with a "logico-philosophical coloration".

Poincaré's response must have come as a surprise to Hilbert, as Poincaré claimed that his original proposition concerning Fredholm's equation included topics from both theoretical physics (i.e., Hertzian waves), and astronomy (i.e., the theory of tides). He agreed to add a lecture on a logico-philosophical topic, stemming from a forthcoming paper on Richard's paradox (Poincaré 1909d).

Hilbert's response to Poincaré's genial proposition has not been located, but he must have been in agreement, because Poincaré wrote back (§ 4.7) with a list of five lectures:

1. On the reduction of Abelian integrals
2. On applications of Fredholm's method
3. The theory of tides and Fredholm's equation
4. Hertzian waves and Fredholm's equation
5. On the notion of transfinite cardinal numbers

Whether Hilbert was pleased with Poincaré's program or not is difficult to know, although he had successfully negotiated the inclusion of topics from theoretical physics and mathematical logic. Poincaré's focus on Fredholm's equation was surely welcomed by Hilbert, who by 1904 had seen therein the possibility of developing a new framework for the study of boundary value problems, which would lead eventually to the concept of a Hilbert space (Archibald & Tazzioli 2014).

The correspondence between Hilbert and Poincaré tells us much we didn't know about Poincaré's lecture series, but it also raises a few new questions. In particular, one wonders why Poincaré added a sixth lecture to the five he announced to Hilbert, and why this sixth lecture was not on Fredholm's equation or mathematical logic, but on the theory of relativity? Perhaps Poincaré took Hilbert's suggestion to add a topic on theoretical physics or astronomy to heart, as this sixth lecture included both.

3 Historical upshot of the Hilbert-Poincaré exchange in 1908–1909

Hilbert's invitation to Poincaré on behalf of the Wolfskehl-Stiftung had multiple objectives. It was meant to reinforce ties between French and German mathematicians, as Hilbert's opening speech made clear from the outset.[2] Beyond this explicit and laudable goal, Poincaré's visit was designed to stimulate research by the members of the Göttingen Mathematical Society, and among German mathematicians in general. Hilbert may have understood the invitation as a way of encouraging Poincaré to take an interest in the ongoing research of GMS members, himself included, via informal exchanges.

On all three of these counts, Poincaré-Week must be rated at least a relative success. The credit here belongs in part to Poincaré, who took the risk of presenting work-in-progress. His lecture on the diffraction of Hertzian waves is one example; the lecture on the new mechanics is another. These two lectures will be discussed in what follows; for an overview of all six lectures, see Gray (2013, 416).

The topic of the propagation of Hertzian waves was one that was well-chosen for Göttingen. Poincaré's interest in Hertzian waves may be dated from his correspondence with Hertz in 1890 (Walter et al. 2007b, § 2-30-1); he lectured on Hertzian waves at the Sorbonne (Poincaré 1894), and was particularly interested in explaining wave propagation over great dis-

[2] Nachlass Hilbert 579, Handschriftenabteilung, Niedersächsiche Staats- und Universitätsbibliothek.

tances (Poincaré 1903). In Göttingen, Max Abraham was a leading expert on Hertzian waves, as was another member of Poincaré's audience, the former assistant to Felix Klein, and since 1906, professor of theoretical physics in Munich, Arnold Sommerfeld. Poincaré may have noticed Sommerfeld's long paper on the propagation of Hertzian waves (Sommerfeld 1909), published in the leading German physics journal *Annalen der Physik* in the first week of February, 1909.

Poincaré had promised Hilbert a lecture on Hertzian waves as an application of Fredholm's method, and the lecture that he delivered in Göttingen fit the bill precisely. His lecture did not stray far from the content of a triplet of notes Poincaré published in the *Comptes rendus* of the Paris Academy of Science on 22 February, 29 March, and 13 April, 1909 (Poincaré 1909b, 1909e, 1909f).

Upon his return to Paris, Poincaré continued to work on the problem of wave propagation, and published again on 7 June (Poincaré 1909c). The published version of Poincaré's lecture thus represents a work-in-progress, up to and including the results contained in the notes published in the *Comptes rendus* on 13 April, one week before the beginning of his Wolfskehl lectures.

The question then arises of the effect, if any, that Hilbert's invitation had on Poincaré's engagement in 1909 with the problem of wireless wave propagation over a curved surface. It is plausible that Sommerfeld's publication renewed Poincaré's interest in the problem, inasmuch as he found it to be a good candidate for the application of Fredholm's method. Hilbert's interest in applying and extending Fredholm's method may have been a consideration in Poincaré's topic choice, as well, but Poincaré does not appear to have been conversant with Hilbert's results in this area.

For example, in Poincaré's first note of the year 1909, entitled "On some applications of Fredholm's method", he acknowledged his neglect of one of Hilbert's results:

> I take this opportunity to make amends for an involuntary omission that Mr. Picard pointed out to me.
>
> In a recent Note, I pointed out a series of results relative, respectively, to the cases in which the kernel of Fredholm's equation becomes infinite of order $< \frac{1}{2}$, $< \frac{2}{3}$, $< \frac{3}{4}$, ...; the first of these results had already been obtained via a different method by Mr. Hilbert.[3] (Poincaré 1909g)

[3]"Je profite de l'occasion pour réparer un oubli involontaire qui m'a été signalé par M. Picard.

Dans une Note récente, j'ai signalé une série de résultats relatifs respectivement aux

Poincaré referred here to his note of 21 December (Poincaré 1908), and to Hilbert's first communication to the Göttingen Academy of Science on "basic features of a general theory of linear integral equations" (Hilbert 1904; reed. 1912).

As for the physical question of Hertzian wave propagation, Poincaré concluded his Wolfskehl lecture on this topic with the observation that intercontinental wireless telegraphy was not ruled out by his mathematical analysis. This was surely a welcome result, given that Marconi had succeeded in sending a signal by wireless from Poldhu to St. John's, Newfoundland in 1901. However, Poincaré soon realized that his analysis was faulty; he corrected himself in a subsequent paper, finished on 15 October, 1909 (Poincaré 1910d), and according to the corrected calculation, long-distance telegraphy was no longer possible! He did not correct the error in the proofs of his Wolfskehl lecture, published the next year (Poincaré 1910a), but appended a short note in French, alerting the reader to his error.

Sommerfeld took note of Poincaré's contribution, and set his student H. W. March the task of using his own approach to solve the same problem, i.e., applying an integral expansion in the case of Hertzian waves propagating over a spherical conductor. March's result disagreed with that of Poincaré, and in March, 1912, Sommerfeld wrote to Poincaré to see if he could find the reason for the divergence. Poincaré wrote back to inform Sommerfeld that he had located the point of divergence: March's integration of Hankel's function was incorrect, due to a defective asymptotic expansion; see Poincaré to Sommerfeld, in Walter et al. (2007b, § 2-54-1). In the note Poincaré communicated to the Paris Academy on this topic, he observed that, once March's error was corrected, the result of March's analysis was identical to his own, such that his earlier result had been "confirmed" by Sommerfeld's student (Poincaré 1912). He remarked further that the latest measurements by Louis Austin off the coast of Virginia of the power of electric-arc-generated wireless waves pointed to a serious disagreement with his theory, such that there was "something here to discover".[4]

It appears that Poincaré-Week facilitated an exchange between Sommerfeld and Poincaré on the topic of wireless wave propagation, which was mutually beneficial. Much the same may be said of the sixth and final lecture of Poincaré-Week, on the new mechanics of relativity.

Poincaré's decision to add a sixth lecture to the program he had announced to Hilbert circa March, 1909, is not easily understood. While

cas où le noyau de l'équation de Fredholm devient infini d'ordre $< \frac{1}{2}$, $< \frac{2}{3}$, $< \frac{3}{4}$, ...; le premier de ces résultats avait déjà été obtenu par une autre voie par M. Hilbert."

[4]Poincaré was right about this; for the subsequent history of long-distance wireless-wave propagation, see Yeang (2013).

Hilbert had specifically requested that Poincaré treat a topic on mathematical physics or astronomy (§ 4.5), his lecture on Hertzian waves surely satisfied this desideratum. Why then did Poincaré choose to speak on the new mechanics in Göttingen?

A tentative answer to this question may be formulated by recalling the dual contexts of relativity theory in Paris and Göttingen.[5] In both places, in the early years of the twentieth century, theorists turned to theories of the electron in order to address the experimental results of electron beam deflection by crossed magnetic fields, magneto-optics, electrodynamics of moving bodies, and black-body radiation. In 1905, Poincaré proposed a modification of Lorentz's electron theory, in which the laws of all physical interactions are governed by covariance with respect to what he called the "Lorentz group". The law of gravitation was clearly a potential spoiler for his theory, but Poincaré showed that as long as the propagation velocity of gravitation is no greater than that of light, this law, too, could be Lorentz-covariant (Walter 2007a).

In order to prove the latter result, Poincaré introduced a four-dimensional space with one imaginary axis, which he used to form Lorentz-invariant quantities. Minkowski noticed this novel method of Poincaré's, and realized that it could be generalized into a four-dimensional vector formalism. Furthermore, the geometry of phenomenal space could be taken to be the geometry of these four-dimensional vectors. He announced this new "spacetime" with great pomp at the meeting of the German Association in Cologne, in September, 1908 (Walter 2008, 2010).

Although Minkowski had earlier acknowledged Poincaré's fundamental contribution to relativity theory, he failed to do so in his Cologne lecture, prompting a worried reaction from Poincaré's allies (Walter 1999). As mentioned above, Minkowski had no further occasion to characterize Poincaré's contribution to relativity theory, as he died in January, 1909. The printed version of his Cologne lecture appeared three weeks after his death (Minkowski 1909), and it is not unlikely that Poincaré had the occasion to read it before delivering his Wolfskehl lectures.

Poincaré's lecture on the new mechanics bears no explicit reference to the work of Einstein or Minkowski. The order of arguments resembles that of the plenary lecture, longer and more detailed than the Wolfskehl talk, that Poincaré delivered on 3 August to the French Association for the Advancement of Science in Lille (Poincaré 1909a). The latter circumstance does not explain fully why Poincaré broached the topic of relativity in Göttingen in

[5] For an overview of research on relativity in Paris, see Walter (2011), and for research in Göttingen, see Pyenson (1979), Walter (1999), and Corry (2004).

April, 1909.

I'd like to suggest that in the early months of 1909, Poincaré realized the potential sway of the Einstein-Minkowski theory of relativity, and sought to defend his own theory in the city that was just beginning to be identified with the relativity revolution. As an indicator of this identification, recall that the *second* Wolfskehl lecture series was assigned to another founder of relativity theory, H.-A. Lorentz, who prefaced his remarks on relativity with the following words:

> It is a particularly welcome task for me to discuss the Einsteinian principle of relativity here in Göttingen, where Minkowski worked.[6] (Lorentz 1913, 74)

Lorentz went on to mention two other Göttingen scientists who had contributed powerfully to the construction of the "mathematical side" of relativity theory: Max Abraham and Arnold Sommerfeld, both of whom attended Poincaré's lectures in 1909.

In addition to the above considerations of place and time, the content of Poincaré's Wolfskehl lecture on new mechanics features two conceptual novelties, which may be linked to these considerations. Prior to Poincaré-Week, Poincaré had promoted a view of relativity in which clocks are always at rest with respect to the ether. This view stands in contrast to the theories backed by Einstein and Minkowski, in which clocks in uniform motion are not distinguished from clocks at rest. The "proper time" *(Eigenzeit)* of a particle in motion, in Minkowski's spacetime theory, is just the time read by a comoving ideal clock, and this time will differ in general from the time read by non-comoving ideal clocks.

Once in Göttingen, Poincaré decided to allow clocks to move. In order to preserve the principle of relativity, this meant that the time read by clocks in motion is deformed with respect to the time read by clocks at rest with respect to the ether. To drive home the idea of time deformation for his audience, Poincaré introduced two observers A and B in relative motion, equipped not just with timekeepers, but with wireless transmitters and receivers. In keeping with his third lecture on wireless wave propagation, Poincaré equipped his observers with the means of transmitting time-stamped position data on the fly. In spite of this high-technology equipment, Poincaré's comoving observers were still unable to detect their absolute motion:

> A can believe he is at rest, and B's apparent speed will be 400000 km/s. If A knows the new mechanics he will say to himself: "B

[6] "Die Einsteinsche Relativitätsprinzip hier in Göttingen zu besprechen, wo Minkowski gewirkt hat, erscheint mir eine besonders willkommene Aufgabe."

> has a speed that he cannot attain, so it must be that I, too, am in motion." It seems that he could determine his absolute situation. But he would have to be able to observe B's motion. To make this observation, A and B begin by setting their watches, then B sends telegrams to A indicating his successive positions; putting these signals together, A can give an account of B's motion, and trace its curve. Well, the signals propagate at the speed of light; the watches marking apparent time vary at every instant and it all will go down as if B's watch were fast.[7] (Poincaré, 1910b, 54–55)

The tabulation of telemetric data would, in principle if not yet in practice, show that the watches of the two observers in relative motion did not run at the same rate.

In fact, in the circumstances described by Poincaré, relativity requires that B's watch retard with respect to that of A. The sign error notwithstanding, Poincaré's Wolfskehl lecture on the new mechanics was his first-ever invocation of the deformation of time due to translation (Walter 2014).

In summary, Poincaré's Wolfskehl lectures on the new mechanics and on Hertzian wave propagation reflect a possible awareness on his part of recent advances in these areas by Minkowski and Sommerfeld, respectively. In light of the subsequent history of these two topics, both Hilbert and Poincaré had reason to be satisfied with the lecture series. Their epistolary exchange in 1908–1909 gives us a better idea of Hilbert's motivation in inviting Poincaré to Göttingen, and of Poincaré's intentions in accepting the invitation.

[7] "A peut se croire au repos et la vitesse apparente de B sera, pour lui, 400000 kilomètres. Si A connaît la mécanique nouvelle il se dira: B a une vitesse qu'il ne peut atteindre, c'est donc que moi aussi je suis en mouvement. Il semble qu'il pourrait décider de sa situation absolue. Mais il faudrait qu'il puisse observer le mouvement de B lui-même; pour faire cette observation A et B commencent par régler leurs montres, puis B envoie à A des télégrammes pour lui indiquer ses positions successives; en les réunissant, A peut se rendre compte du mouvement de B et tracer la courbe de ce mouvement. Or les signaux se propagent avec la vitesse de la lumière; les montres qui marquent le temps apparent varient à chaque instant et tout se passera comme si la montre de B avançait."

4 Annex. The Hilbert-Poincaré correspondence, 1908–1909

4.1 Hilbert to Poincaré

Göttingen d. 6.11.08

Sehr geehrter Herr Kollege

Ihre Zusage hat uns alle hoch erfreut und auch in der mathematischen Gesellschaft, in der ich gestern Ihren Brief mitteilte, wurde allgemein Freude ausgedrückt.

Was nun die Zeit Ihres Herkommens betrifft, so möchte wir als das Optimum bezeichnen, wenn Sie Ihre Vorträge innerhalb der Zeitraumes

27 Febr. bis 10 März

verlegen könnten; allenfalls liesse sich dieser Spielraum noch um einige Tage am Anfange und Ende erweitern. Sollte Ihnen diese Zeit nicht möglich sein, so müssten wir die letzte Aprilwoche (Anfang des Sommersemesters) in Aussicht nehmen.

Vorbereitungen unsererseits bedarf es ja nicht; aber, da wir die Zeit, sowie die Gegenstände Ihrer Vorträge gern zeitig genug in den *Jahresberichten der Deutschen Mathematikervereinigung* bekannt machen und auch unseren auswärtigen Freunden und Kollegen mitteilen möchten, so bitte ich Sie um Mitteilung Ihrer Entschlüsse, sobald Ihnen dies möglich ist.

Mit den besten Grüssen

Hochachtungsvoll und ergebenst

Hilbert

ALS 2p. Private collection, Paris 75017.

4.2 Poincaré to Hilbert

[Between 6 and 18.11.1908]

Mon cher Collègue,

Je suis très flatté de votre proposition et je suis très disposé à l'accepter. Seulement il y a un obstacle. Je ne sais si je serai libre à l'époque que vous fixez.[8] L'Académie française n'a encore choisi ni le jour de ma réception, ni

[8] Hilbert a suggéré par lettre à Poincaré du 25.02.1909 que la série de conférences ait lieu entre le 27.02 et le 10.03.1909 (§ 4.1).

celui des élections. Mais tout fait prévoir que ce sera à la fin de février ou au commencement de mars.[9]

Pourriez-vous me dire entre quelles limites on pourrait faire varier la date de mon voyage à Göttingen ; si au besoin on pourrait le remettre au semestre d'été, et à quel moment il convient que je vous donne une réponse définitive.

Veuillez agréer, mon cher Collègue, l'assurance de mes sentiments affectueux et de mon admiration pour votre talent. Seriez-vous assez bon pour me rappeler au souvenir de M. Klein.

Votre bien dévoué Collègue,

Poincaré

ALS 2p. Cod. Ms. D. Hilbert 312, Niedersächsische Staats- und Universitätsbibliothek, Handschriftenabteilung.

4.3 Hilbert to Poincaré

Göttingen den 19 Nov. 08.

Sehr geehrter Herr Professor.

Wir rechnen nun darauf, dass Sie Ihre Vorträge in die Woche vom 22–28sten April nächsten Jahres verlegen, da diese Tage für uns wegen des Beginnes der Sommersemester die beste Zeit sind.[10] Ich sehe mit grossem Interesse der Mitteilung Ihrer Programmes entgegen.

Mit ergebensten Grüssen

Ihr

Hilbert

ALS 1p. Cod. Ms. D. Hilbert 312, Niedersächsische Staats- und Universitätsbibliothek, Handschriftenabteilung.

4.4 Poincaré to Hilbert

[After 19.11.1908]

Mon cher Collègue,

Voici les titres des sujets que je me propose de traiter.

Sur quelques applications de la méthode de Fredholm.

[9]Poincaré souhaitait éviter un conflit d'emploi du temps avec sa réception à l'Académie française. Sa réception a eu lieu le 28.01.1909, alors que la série de conférences Wolfskehl a eu lieu du 22 au 28 avril, 1909 (Poincaré 1910c).

[10]In his previous letter to Hilbert (§ 4.2), Poincaré informed his German colleague that his upcoming reception at the *Académie Française* conflicted with the period Hilbert had proposed for the lecture series.

Sur la réduction des intégrales abéliennes.

Je suppose que je reste libre de modifier ce programme s'il y a lieu.

Je serai très heureux d'avoir l'occasion de vous voir.

Veuillez transmettre mes compliments à M. Klein et croire à ma sincère amitié et à mon entier dévouement,

Poincaré

ALS 2p. Cod. Ms. D. Hilbert 312, Niedersächsische Staats- und Universitätsbibliothek, Handschriftenabteilung.

4.5 Hilbert to Poincaré

Göttingen den 25.2.09

Hochgeehrter Herr Kollege,

Wie ich Ihnen schon mitzuteilen mir erlaubte, beabsichtigen wir zu der Göttinger ‚Poincaré-Woche' 22–28 April, auch einige Nicht-Göttinger Mathematiker heranzuziehen. Würde es Ihnen vielleicht möglich sein, auch ein Thema aus der mathematischen Physik oder der Astronomie und ein solcher Logisch-philosophischer Färbung zu behandeln? Wir könnten in diesem Falle auch die betreffenden Göttinger Fachkollegen zu Ihren Vorträgen einladen.

Auch beabsichtigen wir an einem oder anderen Abend jener Woche eine Sitzung der hierigen mathematischen Gesellschaft abzuhalten, wo wir dann unsererseits nach unseren Kräften etwas zum Besten geben könnten.

Endlich ist für den 30sten April, dem Geburtstage von Gauss, in dem benachbarten Dransfeld auf dem „hohen Hagen" (der einen Ecke des Gaussischen geradlinigen Dreieckes, für welches er die Winkelsumme π beobachtet hat) die Einweihung einer Gaussturmes projektiert. Ihre Anwesenheit dabei wäre dringend wünschenswert.[11]

[11] The cornerstone-laying ceremony of the Gauss monument was scheduled to take place on the anniversary of Gauss's birthday, on April 30, 1909 (see the notice in the *Jahresbericht der deutschen Mathematiker-Vereinigung* 17, 1908, 121).

According to a story popular in Göttingen at the time, Carl Friedrich Gauss (1777–1855), director of the Göttingen Observatory, and professor of mathematics at the University of Göttingen, tested the Euclidicity of space in the 1820s, by employing his heliotrope to measure the angle sum of a triangle formed by the mountaintops of Brocken, Inselsberg and Hohenhagen (Scholz 2004). In 1908, Felix Klein solicited donations from astronomical and mathematical societies around the world, in order to build a tower on the Hohenhagen commemorating the work of the eminent Göttingen geometer.

Poincaré accepted Hilbert's invitation to attend the cornerstone-laying ceremony (§ 4.6). His presence at the inauguration of the Gauss monument was poignant, in light of what Eduard Study (1914, 117) later called the polemic between Poincaré and the writings of Gauss, Riemann and Helmholtz. For these authors the geometry of space was in some sense empirically determined, a position contested by Poincaré.

Leider sind wir – ganz besonders aber ich – durch den vor kurzem erfolgten Tod Minkowski's in tiefe Trauer versetzt. Ich habe an ihm meinen liebsten und treuesten Jugendfreund, der mir tausendmal mehr wie ein Bruder war, ganz plötzlich und jäh verloren (durch Blinddarm-Entzündung). Es war ein Schlag aus dem heitersten Himmel.[12]

Mit den besten Grüssen

Hochachtungsvoll

Hilbert

ALS 3p. Collection particulière, Paris 75017.

4.6 Poincaré to Hilbert

[Après le 25.02.1909]

Mon cher Collègue,

Mon programme sur les applications de l'équation de Fredholm comprend des applications à la Physique Mathématique et à l'Astronomie, en particulier à l'étude des marées et à celle des ondes hertziennes.[13] Je pourrais aussi, si vous le désirez, prendre comme sujet relatif aux ensembles, une note qui va prochainement paraître dans les *Acta Mathematica*.[14]

Je pourrai assister à l'inauguration de la tour de Gauss.

Je suppose que je puis faire mes conférences en français; s'il en était autrement, je pourrais m'en tirer, mais je vous prierais de m'en avertir un certain temps d'avance.

Votre bien dévoué Collègue,

Poincaré

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4.7 Poincaré to Hilbert

[Ca. 03.1909]

Mon cher Collègue,

Merci de votre lettre. Nous pourrions alors prendre pour titres des diverses communications.

[12] Hermann Minkowski (1864–1909) died in Göttingen on 12.01.1909.

[13] Poincaré répond au souhait exprimé par Hilbert (§ 4.5) qu'il augmente son programme, en traitant une question de physique mathématique ou d'astronomie, et et en abordant un sujet logico-philosophique.

[14] Il s'agit des remarques sur le paradoxe de Jules Richard; voir Poincaré (1909d).

Sur la Réduction des Intégrales Abéliennes.

Sur quelques applications analytiques de la méthode de Fredholm.

La théorie des Marées et l'équation de Fredholm.

Les ondes hertziennes et l'équation de Fredholm.

Sur la notion de nombre cardinal transfini.[15]

Maintenant il y a un point sur lequel je désire attirer votre attention. Je suis encore sous le coup de l'accident qui m'a frappé l'année dernière à Rome et je suis impérieusement obligé à certaines précautions. Je ne puis boire ni vin, ni bière, mais seulement de l'eau. Je ne puis assister à un banquet, ni à un repas prolongé.[16]

Cette circonstance m'avait fait hésiter à accepter votre invitation, mais j'ai pensé que vous sauriez arranger les choses en conséquence.

Je pense qu'il y a moyen de voir nos collègues dans d'autres circonstances que dans des banquets et j'espère dans ces conditions, avoir le plaisir de faire leur connaissance. Je serai enchanté en particulier d'avoir l'occasion de vous voir.

Votre bien dévoué Collègue,

Poincaré

ALS 2p. Cod. Ms. D. Hilbert 312, Niedersächsische Staats- und Universitätsbibliothek, Handschriftenabteilung.

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[15]Poincaré a prononcé des conférences sur les cinq sujets annoncés ici, ainsi que sur un sixième sujet, intitulé "La mécanique nouvelle." Uniquement cette sixième conférence sera publiée en français, les autres paraîtront en allemand; voir Poincaré (1910c).

[16]Lors du Congrès international des mathematicians tenu à Rome en avril 1908, Poincaré a eu une malaise, liée alors par les médicins à une hypertrophie du prostate; voir Darboux (1916, LXVI).

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