

The Poincaré Pear and Poincaré-Darwin Fission Theory in Astrophysics, 1885-1901

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Résumé : Henri Poincaré a découvert dans les années 1880 une figure d'équilibre d'une masse fluide en rotation uniforme : la figure piriforme, ou la poire. Il considérait que la poire était susceptible de se diviser en deux parties inégales, et que son évolution pouvait expliquer la genèse des étoiles binaires. Les études photométriques et spectroscopiques des étoiles variables de cette période ont donné lieu à la modélisation en étoiles binaires à éclipses, ce qui renforçait l'interprétation réaliste des figures d'équilibre – y compris la poire – dans le domaine cosmique. Cet article propose une analyse de l'interprétation astrophysique de la poire de Poincaré et de la théorie de Poincaré-Darwin sur la genèse des étoiles binaires par fission, en regard des recherches sur les étoiles variables réalisées entre 1885 et 1901.

Abstract: In the early 1880s, Henri Poincaré discovered an equilibrium figure for uniformly-rotating fluid masses—the pear, or piriform figure—and speculated that in certain circumstances the pear splits into two unequal parts, and provides thereby a model for the origin of binary stars. The contemporary emergence of photometric and spectroscopic studies of variable stars fueled the first models of eclipsing binaries, and provided empirical support for a realist view of equilibrium figures—including the pear—in the cosmic realm. The paper reviews astrophysical interpretation of the Poincaré pear and the Poincaré-Darwin fission hypothesis with respect to research on variable stars from 1885 to 1901.

1 Introduction: scientific knowledge, far from equilibrium

In the foreword to his exhaustive monograph on variable stars, published in 1924, the Vatican astronomer Father Johan Stein (S.J.) observed that

a straightforward theory of these objects was “virtually unimaginable”.¹ Contemporary observers were agreed. Variable stars had been observed for centuries, and the periodicity of their maximum luminosity tabulated, but it was only with the discovery of spectroscopic binary stars in 1889 that their orbital elements were estimated, and their origins and evolution imagined. Most notably, periodic luminosity variation was largely understood by astronomers to stem from eclipsing binary stars. Just a few years before the first discovery of a spectroscopic binary, Henri Poincaré suggested that binary stars find their origin in the fission of a member of a new series of equilibrium figures: the pear, or piriform figure [Poincaré 1885].

Poincaré’s effort to prove the stability of the pear, and those of Aleksandr Mikhailovich Liapunov to prove the contrary, are well-known in outline to scientists and historians [Jardetzky 1958]. In his history of equilibrium figures, the astrophysicist Subrahmanyan Chandrasekhar wrote memorably of the “grand mental panorama” created by Poincaré’s discovery of the pear and its role in cosmogony, which was “so intoxicating that those who followed Poincaré were not to recover from its pursuit” [Chandrasekhar 1969, 11].

The panorama and intoxication of the Poincaré pear are explored in this paper from an epistemological standpoint that engages with two questions: (1) how did the fission theory come to prominence in astrophysics and cosmology, and (2) what was Poincaré’s own engagement with astrophysical research on eclipsing binary stars? These are the questions I address in some detail for the period beginning with Poincaré’s discovery of the piriform figure in 1885, and ending in 1901, when Poincaré first publicly embraced the emergent theory of eclipsing binaries.

General histories of astronomy typically underline how a class of variable stars, the Cepheids, came to provide a reliable distance gauge both in the Milky Way and beyond, in the clusters and galaxies of our galactic neighborhood. A distinction was not made between eclipsing binaries and Cepheid-type variables until around 1913, when a pulsation model was introduced for the latter. This was an important moment for the eclipse model, as it no longer had to account for the light curves of Cepheid variables. Even so, theorists continued to deploy variants of the eclipse hypothesis to explain Cepheid light curves well into the 1920s.²

My account begins with a brief, non-technical summary of Poincaré’s claim to have discovered a figure of equilibrium, known as the pear, or piriform figure, and its interpretation in astrophysics in the 1890s as a preliminary stage in the

1. “Denn soweit sich das ganze Feld überschauen läßt, sind die physikalischen Ursachen der Veränderlichkeit so verschieden und verwickelt, daß man versucht ist, eine Formulierung, die den Keplerschen Gesetzen an Einfachheit auch nur von weitem gleichkäme, für geradezu undenkbar zu halten” Stein [1924, V].

2. General histories of astronomy with useful accounts of eclipsing binaries include [Pannekoek 1961] and [Herrmann 1984]. Primary sources in astronomical photometry and spectroscopy are well-referenced by Hearnshaw [1996, 2014]. For a brief, non-technical history of variable stars see [Hogg 1984].

genesis of binary stars. The second section takes up George William Myers' model of the variable star β Lyrae as an eclipsing binary, while the third and final section recalls Charles André's presentation of Myers' model, Poincaré's reviews of André's textbook, and a letter from Myers to Poincaré.

2 The Poincaré pear and the Poincaré-Darwin fission theory

Isaac Newton assumed the Earth to be a rotating figure of equilibrium, such that the universal force of gravitation counter-balanced the forces of centrifugal acceleration of the spinning globe. Newton found the consequent figure of the Earth, assuming homogeneity, to be an oblate spheroid.³ Measurement of the variations of gravitational force with terrestrial latitude, and of the apparent form of Jupiter by G.-D. Cassini and John Flamsteed tended to confirm Newton's model, while the mathematical theory was developed by Colin Maclaurin and others in the eighteenth century.

In the first half of the nineteenth century, Carl Gustav Jacob Jacobi showed that, contrary to intuition, there are figures of equilibrium of rotating, homogeneous fluid masses that are *not* figures of revolution, including ellipsoids of *three* unequal axes. In the 1880s, Poincaré took an interest in this mathematical problem, inspired in part by the analysis of the stability of Saturn's rings by Sofia Kovalevskaja, as he explained—anonously—in the pages of the Parisian journal *Le Temps* in 1886. This was also the occasion to announce his own discovery of a new series of equilibrium figures: the apiodal, or piriform figure of equilibrium. His description appealed to the imagination:

Imagine a fluid mass contracting via cooling so slowly that it remains homogeneous, and its angular velocity is the same at every point. From a nearly spherical form it flattens more and more, while remaining an ellipsoid of revolution. Then the equator itself ceases to be circular, becoming elliptical; the fluid mass assumes the form of an ellipsoid with three unequal axes. Next, the median part of the ellipsoid thins out; one of the halves tends to elongate more and more, and the other half tends toward a spherical form. Everything leads us to believe that if the cooling

3. Newton assumed a homogeneous figure of radius R rotating with angular velocity Ω and ellipticity ϵ . Denoting the gravitational constant G and the terrestrial mass M , Newton related ellipticity to the centripetal acceleration at the equator, $\Omega^2 R$, divided by the mean gravitational acceleration at the surface, GM/R^2 , such that $\epsilon = \frac{5}{4} \frac{\Omega^2 R^3}{GM}$. For accounts of Newton's line of reasoning, see [Guicciardini 1989], [Greenberg 1995].

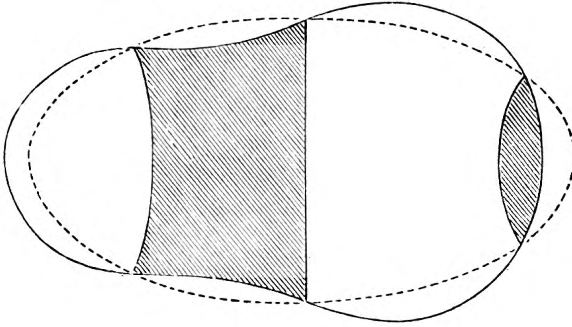


Figure 1: Poincaré's illustration of the piriform figure, from [Poincaré 1885].

were to continue, our fluid mass would divide in two distinct and unequal bodies.⁴

The mathematical theory of the pear motivating Poincaré's thought experiment had recently appeared in the pages of *Acta Mathematica*, along with an illustration (Fig. 1) comparing a contour of his pear to that of a Jacobi ellipsoid, represented by a dashed line.

There are two aspects of Poincaré's discovery that I want to underline. First of all, the identification of a new series of equilibrium figures was a remarkable event in the history of mathematics, placing Poincaré's name alongside that of Jacobi, and giving rise to exchanges with A. M. Liapunov, William Thomson and George Howard Darwin, among others.⁵ Four decades later, Élie Cartan showed that the stability of the pear could not be proven by Poincaré's method [Lyttleton 1953, 4], which leads me to my second point.

From the outset, Poincaré stressed *fission* as a possible outcome for a homogeneous rotating fluid mass. A rotating fluid mass in space could then

4. "Imaginons une masse fluide, se contractant par refroidissement, mais assez lentement pour rester homogène et pour que la vitesse de rotation soit la même en tous ses points. D'abord presque sphérique, elle s'aplatit de plus en plus, en conservant la forme d'un ellipsoïde de révolution. Puis l'équateur lui-même cesse d'être circulaire et devient elliptique, la masse fluide prend alors la forme d'un ellipsoïde à trois axes inégaux. Ensuite l'ellipsoïde se creuse dans sa partie médiane l'une de ses moitiés tend à s'allonger de plus en plus et l'autre à se rapprocher de la forme sphérique. Enfin tout porte à croire que si le refroidissement continuait encore, notre masse se partagerait en deux corps distincts et inégaux" (Henri Poincaré, manuscript of an unsigned article published in *Le Temps* on 5 May, 1886, auctioned in Paris by ALDE, lot n° 292, on 6 May, 2008, edited in [Walter, Nabonnand *et al.* 2016, § 3-48-1], <http://henripoincarepapers.univ-nantes.fr/corresp/indext.php?a=on&id=4016>). All translations are my own, unless otherwise indicated.

5. For a gentle introduction to stability theory in the nineteenth century, with a focus on Poincaré, see [Archibald 2015]. Poincaré's contribution is discussed by Gray [2013, chap. 5].

be considered as a possible progenitor of a binary star, if one were to admit stars are adequately modeled as homogeneous fluid masses. In the state of knowledge of the internal structure of celestial objects in the late nineteenth century, this was not an unreasonable supposition, and it was one made readily by Poincaré and other mathematical astronomers.

The fission theory of the origin of binary stars flatly contradicted Laplace's nebular hypothesis, according to which the planets of the solar system evolved (roughly) via condensation of diffuse matter surrounding and rotating along with the sun. A similar scenario was imagined for the formation of spiral nebulae, fueling speculation about the age of the cosmos, and providing a mathematical foundation for the evolution of the universe. With the emergence of the science of thermodynamics in the 1850s, and the discovery of the second law of thermodynamics, the universe was understood by physicists like W. Thomson to be approaching its demise, the so-called "heat-death of the universe".⁶ Poincaré was fully aware of the contradiction, and he sought to deflate it by noting that the nebular hypothesis concerned a heterogeneous matter distribution, while the fission theory assumed a homogeneous fluid mass [Poincaré 1885, 379].

The fission theory was associated first with Darwin, who was intrigued by the evolution of orbital parameters of the Earth-moon system, including the gradual decrease of the Earth's rotational velocity. Taking into account tidal forces, and working backwards in time, Darwin estimated that these two bodies resulted from fission no less than fifty-four million years ago [Darwin 1880, 882]. As Darwin explained, he was originally motivated to study this problem by the Kant-Laplace nebular hypothesis:

It was in the hope that the investigation might throw some light on the nebular hypothesis of Laplace and Kant that I first undertook the work. It must be admitted, however, that we do not obtain much help from the results. It is justly remarked by M. Poincaré that the conditions for the separation of a satellite from a nebula differ from those of his problem in the great concentration of density in the central body. [Darwin 1887, 442]

Unlike Darwin, Poincaré worked forward in time, while assuming, like Darwin, a homogeneous rotating fluid mass. Poincaré further developed a powerful analytical approach to the study of equilibrium figures building on the contributions of C. G. J. Jacobi, Joseph Liouville, and W. Thomson and Peter Guthrie Tait.⁷

A crucial concept in Poincaré's approach is that of a point of bifurcation in a sequence of equilibrium figures arranged linearly with respect to some

6. On the nebular hypothesis in astronomy and geophysics, see Brush [1996]; on its broader interpretation, see [Beer 1989], [Schaffer 1989].

7. On Liouville's largely unpublished work on equilibrium figures of rotation, see [Lützen 1984].

parameter (e.g., angular velocity), or to a system of parameters. He wanted to show that new equilibrium forms could, for a given parameter, bifurcate from such a sequence, and exchange their stability at the branch point [Mawhin 2014]. James Jeans later considered the two-dimensional case of a rotating infinite cylinder [Jeans 1903], which features several analogies to the three-dimensional case, and as Darwin underlined in his review of research on the genesis of binaries, leads to a pear-shaped equilibrium form [Darwin 1909]. The separate question of the perturbative stability of the pear was of interest to mathematicians and theoretical astrophysicists alike, in a context of competing definitions of stability [Roque 2011].

Double stars had long captured the attention of astronomers, along with variable stars. In the late nineteenth century, aided in particular by more powerful telescopes, and new methods in photography, photometry and spectroscopy, astronomers and astrophysicists learned much about these objects, and began cataloging them according to sky position, magnitude and spectral class. The number of known binary stars grew rapidly when spectral lines of certain stellar objects matched those of the spectra of two stars in distinct spectral classes (or subclasses). Estimates of binary star density in the general stellar distribution grew accordingly. In 1911, Poincaré thought that one in three stars “at least” was binary [Poincaré 1911, 394]; William Wallace Campbell put this figure at one in five or six stars, for the spectral classes F, G, K or M [Campbell 1913, 280]. Knowing something about the origin of binary stars had become a primary objective for astronomy by the second decade of the twentieth century.

Scientific interest in variable stars dates from the early eighteenth century, when the observed variation in magnitude was attributed to (unseen) dark spots, which would recur periodically with rotation of the star. Motion of sunspots offered a concrete local example of this phenomenon. During the final two decades of the century, the York duo of Edward Pigott and John Goodricke—the latter a young deaf-mute gentleman—discovered a number of new variables in Pigott’s private observatory. One of these was Algol (β Persei), the sudden change in magnitude of which prompted Pigott to conjecture, in 1781, the existence of a dark eclipsing companion. News of this discovery was hailed at the Royal Society of London, while William Herschel, who had just discovered the first new planet in recorded history, predicted that the hypothesis of “a plurality of solar and planetary systems” would soon be verified [Hoskin 1979]. The Royal Society lost no time in awarding Goodricke the Copley Medal for the discovery of Algol’s periodic variation in luminosity. In 1786 Goodricke, at the age of 21, and just two weeks before he succumbed to pneumonia, was named a Fellow of the Society.

After the triumph of their discovery of Algol’s periodicity, Pigott and Goodricke naturally continued to seek out new variables, and over the next few years they found three more: η Antinoi (now η Aquilae), β Lyrae and δ Cephei. They recognized that the luminosity variation of these stars did *not* lend itself so immediately to an interpretation in terms of an eclipsing

companion. The first and last of the trio are in fact Type I Cepheid variables. As for β Lyrae, Goodricke correctly characterized its variation over 12.91 days, with *two* maxima of equal luminosity and *two minima*.

The theoretical upshot of the latter discoveries was to render quite doubtful the eclipse hypothesis, even for the explanation of Algol's variation. Pigott seems to have soured on the eclipse theory, as he went on to adopt the view of W. Herschel, according to which observed variation in magnitude results from the rotation of a single dark body covered by a luminous atmosphere of varied thickness. This theory did not invite geometrical modeling at first, and the eclipse hypothesis went dormant for most of the nineteenth century.

Variable stars held intrinsic interest for a few observers in the early nineteenth century, foremost among whom was the Bonn astronomer Friedrich Wilhelm August Argelander. He observed and published the periods of hundreds of variables, including that of β Lyrae, and inspired others to do likewise. Many of Argelander's observations were cataloged in Johann Carl Friedrich Zöllner's *Photometrie des Himmels* [1861], which carefully distinguished known variables.⁸

Zöllner's photometric observations, and his revival of W. Herschel's rotation theory of variables were a source of inspiration for the Swedish mathematical astronomer Hugo Gylden. In 1880, Gylden took up a special case of the rotation theory of variables, by placing constraints on the light curve produced by a body of non-uniform, constant surface luminosity in uniform rotation about its principal axis of inertia. Gylden offered no numerical results, and mentioned only two specific variables: Mira Ceti and β Lyrae. The latter of these, Gylden wrote, was of "little interest from a theoretical standpoint," because its axis of rotation nearly coincided with its inertial axis, "as far as this can now be seen" [Gylden 1880]. In general, until the late 1880s, variables failed to inspire mathematical astronomers.

Visual binary stars, on the other hand, were of some interest to mathematical astronomers. Mathematicians worked with two ellipses: the true path of a companion orbiting the principal star, and its projection on a plane orthogonal to the line of sight, referred to as the apparent ellipse.⁹ A transformation was then sought from the former to the latter. Complications set in with the sensitivity of observations of motion at apparent aphelion and perihelion, which John Herschel (the son of William) sought to manage with probabilistic considerations.¹⁰

The number of known binary stars increased rapidly with the help of spectroscopy. At the Potsdam Observatory, Hermann Carl Vogel (1841-1907)

8. The Leipzig astrophysicist Zöllner is often credited with the foundation of astrophotometry, in light of his design in 1861 of a photometer with a flame reference and a pair of Nicol prisms as the polarizing apparatus [Pannekoek 1961, 385].

9. In the 1890s, astronomers referred to these as the "relative" and "absolute" orbits, respectively.

10. [Herschel 1850]. On the nature of astronomers' interest in double stars, see [Williams 1984], and on J. Herschel in particular, see [Case 2018].

and Julius Scheiner discovered their first *spectroscopic* binary in 1889 [Vogel 1890]. The binary star they observed was one of the variable stars discovered by Goodricke; Vogel noted that the displacement of spectral lines of the brighter component matched the phase of Algol's light curve. Two other spectroscopic binaries were discovered at roughly the same time at Harvard College Observatory (HCO) by Edward C. Pickering and Antonia Maury [Hearnshaw 2014, 53]. In 1893, Vogel was awarded the Henry Draper Medal by the National Academy of Sciences (USA), "for spectroscopic observations upon the motion of stars in the line of sight, and other kindred researches."

Vogel was a pioneer of spectroscopic measurement of radial velocity, and a champion of research on variable stars. His analysis in 1894 of recent observations of β Lyrae declared this variable to be

[...] among the most interesting spectroscopic objects in the northern heavens.¹¹

Vogel's paper took up the spectrograms realized at the Pulkovo Observatory by Aristarkh Belopolsky, and correlated graphically to Argelander's light curve. Belopolsky interpreted the variable as an partially-eclipsing binary with a circular orbit [Belopolsky 1894]. While Vogel did not doubt that β Lyrae is a close binary star, he noted that the light curve was equally consistent with an elliptical orbit, where the major axis is aligned with the line of sight. The real difficulty for Belopolsky's interpretation was the following: at the minima of luminosity, the relative radial velocity of both components must vanish. Similarly, for the luminosity maxima, which ought to correspond to the moments of maximum relative line shift, in opposite directions for the two maxima. None of this, however, was confirmed by Belopolsky's spectrograms. Vogel emphasized the difficulty of reconciling photometric and spectroscopic observations of β Lyrae. Astronomers love a good challenge, and they were well served here by Vogel. Determining the orbital parameters for a system compatible with photometric and spectroscopic observations, and with other constraints from physical considerations became what Stein later called the " β Lyrae problem" [Stein 1924, 379].

Along with Vogel, Pickering was another decisive figure in the rise of variable-star research. As HCO director, Pickering published the first spectroscopic study of Algol, realized from photographs taken over four years, in collaboration with Williamina Fleming and Antonia Maury. The spectrum of β Lyrae, Pickering announced, "is unlike that of any other star hitherto examined" [Pickering 1891]. He interpreted the recorded line shifts as the signs of a close binary with components of unlike spectra, but he was open to two other possibilities. One of these was a "meteor stream", understood to

11. The original sentence reads: "Der veränderliche Stern β Lyrae, bemerkenswerth durch die eigenthümliche Form seiner Lichtcurve, zählt in spectralanalytischer Beziehung zu den interessantesten Objecten des nördlichen Himmels" [Vogel 1894c]. Vogel's paper was translated to English; see [Vogel 1894a,b]. The given translation is borrowed from the latter publication.

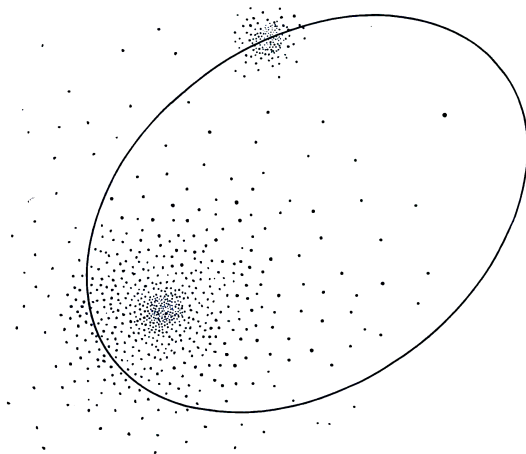


Figure 2: Illustration of two luminous meteor swarms in Keplerian orbit [Lockyer 1890, 479].

be an eclipsing swarm; more on this will follow below. The other possibility was a single rotating sun-like star with a large protuberance which, due to the observed periodic doubling of lines, had to extend beyond the stellar equator.

The discoveries by Vogel, Pickering and others put the eclipse hypothesis back into play for the explanation of short-period variables. A variant of sorts of Pigott’s eclipse hypothesis was put forth at this time by Norman Lockyer (1836-1920), the self-taught astronomer, founder and editor of *Nature*, and from 1877, first director of the Solar Physics Observatory in South Kensington [Hearnshaw 2014, 53]. Lockyer supposed short-period variables to be eclipsing, interpenetrating meteor swarms, where the heat of collision gives rise to vapor. In fact, this was just the latest avatar of the all-embracing “meteoritic hypothesis” he had advanced in 1887, and from which he derived his scheme of stellar evolution. The scheme evolved over time, but the basic idea remained the same: stellar evolution followed a temperature curve arranged according to rising or falling temperature, with the young stars increasing in temperature, changing spectral class along the way, reaching a maximum, and decreasing in temperature with advancing age. Lockyer’s friend George Gabriel Stokes found the meteoritic hypothesis to beg the question of the origin of crystals in meteors, but as an explanation of short-period variables, the image of eclipsing meteor swarms had a degree of plausibility. Pickering found it useful for understanding the line shifts of β Lyrae, as noted above.¹²

12. Under Lockyer’s hypothesis, collision of meteors occurs between two or more companion swarms, and also between the latter and swarms passing through interstellar space; for a more complete presentation, see [Meadows 2008, 200].

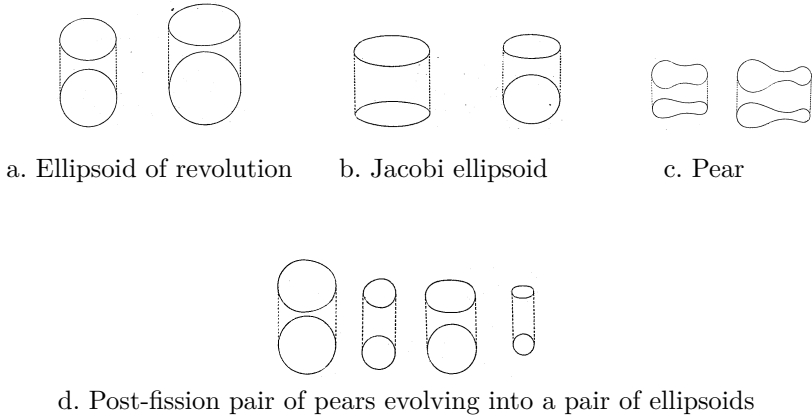


Figure 3: Evolution of an equilibrium figure, from [Poincaré 1892]. The temporal evolution of a single rotating fluid mass is illustrated from (a) to (d). Stacked figures are orthogonal projections, such that the axis of rotation points out of the page in the lower figure. In each case, the left figure evolves into the right figure.

Poincaré, too, contributed to the rise of the eclipse hypothesis, both directly and indirectly. In 1892, he published a general account of equilibrium figures, featuring a series of contours illustrating the evolution of a rotating fluid mass which culminates in fission (Figure 3). As for the *physical reality* of such an evolution, Poincaré drew a direct link to close binaries:

Perhaps the process I have just described [...] is closer to the one that produced certain double stars than that from which the solar system emerged. All this remains very hypothetical in any case.¹³

Poincaré directed the attention of his readers to the deployment of his fission hypothesis beyond the solar system, while not ruling out its use closer to home. As we shall see in what follows, over the next ten years, Poincaré revised his view, and came to regard fission of a self-gravitating, rotating star as the most probable explanation of binary formation.

We can chart the growing interest in variables from 1888 to 1896 based on Seth Chandler's three catalogs. In his presentation of the catalogs, Chandler insisted on the rigorous nature of his selection, while promoting additional discovery efforts. For example, the third catalog reports the recent discoveries of a total of 153 variables in seven globular clusters by Solon Bailey at the Arequipa Observatory, confirmed by Pickering and Fleming. Chandler omitted these newly-discovered variables from his third catalog,

13. "Peut-être le processus que je viens de décrire [...] se rapproche-t-il plus de celui qui a produit certaines étoiles doubles que de celui d'où est sorti le système solaire. Tout dans tous les cas reste très hypothétique" [Poincaré 1892, 813].

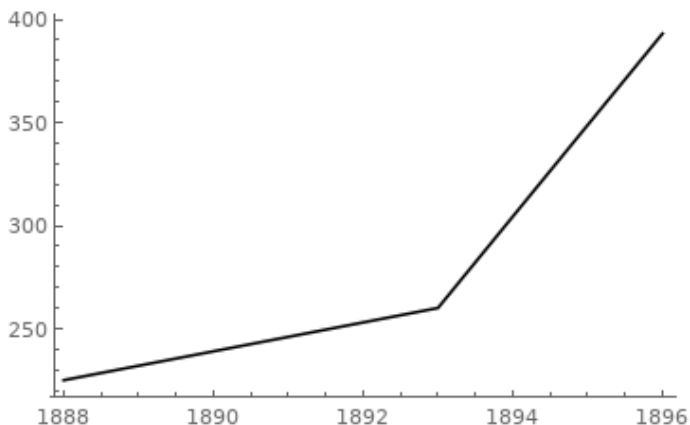


Figure 4: Discovery rate of variable stars, from S.C. Chandler's three catalogs, 1888-1896.

as he did not yet have all the details he required, and he did not wish to delay publication [Chandler Jr. 1888, 1893, 1896]. The task of cataloging new variables soon became overwhelming, and Chandler decided to hand it over to the *Astronomische Gesellschaft*. This was surely a wise move on his part, as the announced discovery of variables in clusters was only the beginning of a torrent of discoveries. Annie Jump Cannon's catalog of 1907 featured 1425 variables, of which roughly a third were found in globular clusters [Müller & Hartwig 1918, IV].

3 G. W. Myers' models of β Lyrae and U Pegasi

The first mathematical astronomer to pick up Vogel's challenge was a young American: George William Myers (1864-1931). Largely forgotten in the general history of astronomy, if not to Kuiper [1941], Struve [1958] and others who study variables, Myers studied engineering at the University of Illinois. Like many other American students in the exact sciences at the time, he went

to Germany to obtain a Ph.D.¹⁴ In 1896, he defended a thesis in theoretical astronomy at the University of Munich on the variability of β Lyrae.¹⁵

Myers' thesis advisor, and director of the Munich Observatory, Hugo von Seeliger suggested that he work out β Lyrae's orbit. The choice of topic was likely inspired in part by Seeliger's own career. As a young man, Seeliger wrote his thesis in Leipzig on the orbits of binaries, and was employed from 1873 to 1878 by Argelander at the Bonn Observatory [Wilkins 1927, 4]. More recently, Vogel's study had shown the β Lyrae problem to be both interesting and challenging, perhaps even impossible. Seeliger had previously charged another doctoral student, Carl Harting, with the construction of a model of the orbit of Algol [Harting 1889]. And while Pickering [1880] made progress with the latter star based on its light curve, and found it to be an eclipsing binary, Harting was the first to determine a differential light curve from a linear variation of orbital parameters [Stein 1924, 283]. Of course, Harting did not take into account any measurements of radial velocity, as these were only obtained for Algol the year after his thesis defense.

The spectrographic method employed by Pickering, Vogel, Maury and others held real promise as a tool for discovering binary stars, and for providing the elements needed to calculate their orbit. In the new class of spectroscopic binaries, a subclass of variables emerged, and soon engulfed all variable stars, as variables were generally assumed to be eclipsing binaries. This remained true even as late as 1918, as Robert Aitkens' monograph shows. At the University of Illinois Observatory, Joel Stebbins, for example, expressed the common-sense view to the effect that "all spectroscopic binary stars would be eclipsing variables for observers properly situated in space" [Stebbins 1911].

By the mid-1890s, several astronomers had produced light curves for the variable β Lyrae. Myers used one with 1439 measurements realized over a span of nineteen years by Argelander, and published in 1859. The idea for Myers was the same as for Vogel: to tune the parameters of his geometric model to obtain a close representation of both the spectroscopic velocity-curve and the photometric light curve. As established by Argelander, β Lyrae's luminosity

14. For an overview of the emergence of mathematical research in the United States, see Parshall & Rowe [1994].

15. From 1888 to 1900, Myers held several positions in succession at the University of Illinois, from instructor of mathematics to assistant, associate, and full professor of mathematics and astronomy [Marquis-Who's Who 1943, 885]. Upon completion of his thesis in Munich, he directed the new observatory in Urbana. In 1900, Myers became head of astronomy and mathematics at the Chicago Institute, an institution for educating future schoolteachers founded by Francis Wayland Parker (1837-1902), a pioneer of the progressive school movement in the United States. In 1901, the Chicago Institute became the School of Education of the University of Chicago [Dewey 1902], where Myers was named professor of mathematics education and astronomy, a position he held until his retirement in 1929. Myers was a member of the astronomical societies of Germany, France, Belgium, Mexico and South America, and belonged to the American Academy for the Advancement of Science, and the American Mathematical Society.

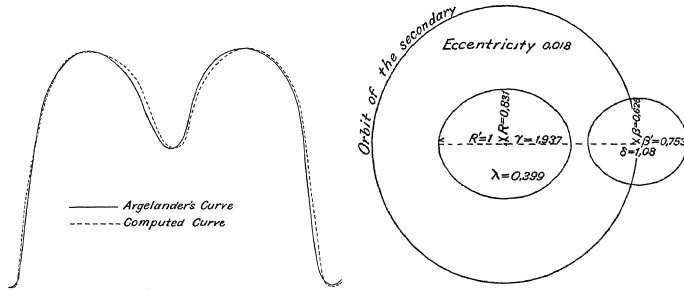


Figure 5: Observed and theoretical light curves, and theoretical orbit of β Lyrae, from [Myers 1898a]. Orbital eccentricity is small, and the axes of the ellipsoids are always aligned.

varied continuously over a period of about 12.91 days, with two roughly-equal maxima and two unequal minima. Myers reasoned that orbiting spheres would never deliver the target light curve, and so he worked with two ellipsoids, one a satellite in circular orbit, and both revolving with their major axes in perfect alignment. When the major axes of the true system are perpendicular to the line of sight, we then have the two maxima. From such geometric relations, Myers determined the ratio of respective ellipsoid axes that best fit the light curve at maxima. He then iterated the orbital elements (eccentricity, longitude of periastron, inclination with respect to line of sight) until he obtained a close fit. The result of Myers' labors was a very close binary, where the distance of the ellipsoid centers is only about 2.4 times the semi-major axis of the larger ellipsoid [Myers 1896, 1898a].

In order to convince himself of the correctness of his method, Myers explained, he perturbed an orbital element, and computed the corresponding change to the light curve. By doing so, he noticed that an orbital solution for which the ellipsoids *interpenetrated* produced a light curve with “nearly the same” probable error as that of his final result. From this circumstance, Myers concluded that β Lyrae was either “not yet separated”, or of recent separation, and consequently,

[...] we seem to have here the first concrete example of a world in the act of being born. [Myers 1898a, 17]

Having concluded the task of orbital determination from the light curve, Myers went on to consider the spectrograms obtained by Belopolsky for β Lyrae in 1892 at the Pulkovo Observatory (mentioned above). Myers established a radial velocity curve from these spectrograms, from which the orbital elements were readily determined using the method Arthur

Rambaut [1891] successfully applied to Pickering's observations of the binary β Aurigae.¹⁶

In order to determine the inclination of the orbit, Myers reasoned that from the form of the light curve, the ellipse had to lie on a plane intersecting the sun. The absolute orbit was then fixed, and Myers proceeded to work out the relative orbit. To do so, Myers relied on an observation of the relative displacement of three dark lines communicated privately by Lockyer, which he converted to velocities, and corrected for terrestrial motion. By calculating the corresponding relative velocities from the absolute orbit, Myers found the ratio of absolute to true velocities to be $1/3.168$, which is just the ratio of major axes, a/A , and also that of the two masses, $m/(m + M)$. Since Myers had no way of knowing which of the masses, m or M , produced Belopolsky's lines, he made an educated guess, such that the mass of the smaller star was about 9.5 solar masses and the larger, about 21 solar masses.¹⁷ From the light curve analysis, Myers knew the volume of the ellipsoids, and he estimated the mean density of the system to be less than that of air. This indicated a nebular system, which prompted Myers to comment:

It appears then that β Lyrae furnishes us a concrete illustration of the actual existence in space of a Poincaré figure of equilibrium.
[Myers 1898*a*, 19]

Remarkably, Myers concluded his thesis with the observation that β Lyrae was not a *binary* system at all. Instead, the combined photometric and spectroscopic observations of the system indicated that it was a *single* spinning body—a body whose form, he wrote, had been studied mathematically by Poincaré and Darwin.

After the successful defense of his thesis in Munich, Myers returned to Urbana, where he was promoted to full professor of astronomy and mathematics, and director of the new observatory.¹⁸ On October 20, 1897, he presented his theory of β Lyrae to astronomers gathered in Williams Bay, Wisconsin, for the inauguration of the Yerkes Observatory. His paper, entitled “The system of β Lyrae”, appeared in the recently-founded *Astrophysical Journal*, which assured its diffusion among astrophysicists. In his paper's conclusion, Myers reiterated and strengthened his interpretation of β Lyrae as a spinning pear:

In conclusion, let it be observed that an attempt at a formal representation of the condition of things prevailing in the system of β Lyrae, leads to the assumption of a single body (such as

16. Useful overviews of models of the motion of binary stars are provided by [Hepperger 1910], [Aitken 1918], [Henroteau 1928].

17. Myers' value for the system mass of β Lyrae is high by current estimate, which puts it at roughly sixteen solar masses (SIMBAD).

18. Myers directed the University of Illinois Observatory until 1900, when he moved to the Chicago Institute [Marquis-Who's Who 1943, 885].

Poincaré's or Darwin's figures of equilibrium). The above has, of course, only a formal significance, but on account of the poverty of observational material at my disposal an attempt to push the discussion farther on a mathematical basis could not have proved profitable. It is believed, however, that the discussion may help us to orient our views with regard to this wonderfully interesting star. [Myers 1898*a*, 22]

Myers took care here to associate his model with the pear, and to suggest that further work was required, both from mathematicians and observers.

The inauguration of the Yerkes Observatory, organized by its director, George Ellery Hale, was arguably the astronomical event of the year for North American astronomy. In attendance for the three-day meeting that preceded the inauguration were sixty-odd physicists and astronomers, in a Who's Who of turn-of-the-century astronomy and astrophysics, including the Göttingen spectroscopist Carl Runge, James E. Keeler (who delivered the keynote address), Albert A. Michelson, George W. Ritchey and Forest R. Moulton. Myers' talk was featured in an afternoon session, along with others by Simon Newcomb, Pickering, Edward E. Barnard, George W. Hough and Father Hagen.¹⁹ The publicity afforded Myers' results on β Lyrae by the Yerkes meeting, and their subsequent publication helped consolidate acceptance of the Poincaré pear and Poincaré-Darwin fission theory in astrophysics.

In the wake of the Yerkes inauguration, Pickering, impressed by Myers' results, asked O. C. Wendell to establish a new light curve for U Pegasi. Originally thought to be an Algol-type eclipsing binary with a very short period of 4.5 hours, this classification was contested, and Pickering hoped to settle the matter with new photometric data.²⁰ On 28 December, Wendell recorded two unequal minima, such the period of the star was doubled, and the resulting light curve of U Pegasi, Pickering wrote, "closely resembled that of β Lyrae" [Pickering 1898].

Pickering did not attempt to work out the orbit of U Pegasi, but instead invited Myers to do so, extending him the use of the HCO photometer. Once he had verified the difference in the observed magnitude in minima found by Wendell, Myers applied his method to determine the probable orbit of U Pegasi from the HCO light curve. This was somewhat simpler than calculating the orbital parameters of β Lyrae from Argelander's light curve, in that satisfactory results were obtained from an assumption of eclipsing spheres, without having to resort to ellipsoids of revolution.

19. Yerkes Observatory program, University of Chicago Library, Special Collections Research Center; undated, page 4. On the inauguration, see [Osterbrock 1999]. At least eighty people were in attendance for the inauguration; many were captured in a photo reproduced in [Osterbrock 1997].

20. Pickering noted Chandler's suggestion that U Pegasi represented a new class of variable, distinct from both the Algol type and that of the short period variables η Aquilae and δ Cephei [Pickering 1898].

The result of Myers' analysis of U Pegasi's light curve will come as no surprise, given Pickering's identification of it (in January, 1898) as a β Lyrae class variable. Pickering invited Myers to present his results at the Harvard Observatory Astronomical Conference held in the drawing room of the director's residence on 19 August, 1898.²¹ Myers announced that, not only is U Pegasi's light curve "satisfactorily represented by the satellite theory", but in addition, it is probably a pear:

The distance of centers does not materially differ from the sum of the radii of the components, suggesting the probable existence of the "apiodal" form of Poincaré. [Myers 1898*b*, 172]

With much encouragement from the HCO director, Myers had now tentatively identified a second variable as a Poincaré pear.²²

Myers was not the only one to endorse the pear, and to latch on to the fission theory: Karl Schwarzschild, Myers' fellow doctoral student in Munich did the same. Schwarzschild defended his doctoral thesis under Seeliger's direction in 1896, on the topic of Poincaré's theory of rotating fluid masses [Schwarzschild 1898]. Much like Myers, Schwarzschild expressed his readiness to tread where Poincaré would not, but his reasoning followed a different path, one indicated by the branching sequence of equilibrium forms:

We don't yet know exactly what is happening here. Probably, as chance will have it, a smaller part of the mass emerges from the ellipsoidal form at one end or the other of the major axis, and the whole mass assumes the pear-shaped form of Poincaré's figures, after which there will be an ever-stronger indentation, and presumably in the end a splitting of the mass into two unequal parts. [...] Mr. Poincaré considers it too bold to want to infer from this history of an invariably-homogeneous mass the reform of Laplace's inhomogeneous nebula. However, if one were to imagine not a gaseous mass, but a liquid one which, even with vanishing surface pressure, always has finite density, [...] then one may conclude that such a liquid mass undergoing increasing contraction will lose its rotational form, and eventually split.²³

21. Among the 93 registered attendees of the "Second Annual Conference of astronomers and astrophysicists" were Hale, Newcomb, Stein (visiting from Leiden), Charles St. John, Benjamin Peirce and the HCO staff; most attendees were from the northeast coast of the United States. For the full attendance list of the three-day meeting, see [Hale 1898].

22. Later studies would bear out the general classification, although U Pegasi was eventually reclassified as an eclipsing binary of a third type, unknown in 1898: W UMa. For a more recent study, see [Djurašević, Rovithis-Linaniou *et al.* 2001].

23. "Was hier geschieht, wissen wir noch nicht genau. Wahrscheinlich tritt ein kleinerer Teil der Masse, wie es wieder der Zufall will, am einen oder andern Ende der grossen Axe aus der ellipsoidischen Form heraus, und die ganze Masse nimmt die birnförmige Gestalt der Poincaré'schen Figuren an, worauf eine stärkere und

Like Poincaré, Schwarzschild viewed the pear as a binary precursor, and found Poincaré's fission theory to be preferable to Laplace's nebular hypothesis as an explanation of binary genesis, at least in the absence of certain knowledge of the internal structure of stars. The latter topic would interest Schwarzschild and others in the coming decade, but at the turn of the century, this was not yet an active field of research in astrophysics.

4 Charles André's *Traité d'astronomie*

The most extensive review of Myers' theory of β Lyrae appeared in French, in the second volume of Charles André's *Traité d'astronomie*. A former student of the *École normale supérieure* who was certified by the *agrégation* in physics, André (1842-1912) was employed by the Paris Observatory in 1865 as an *aide-astronome*. He was not appreciated by the director, Le Verrier, but was recruited to a chair in physical astronomy by the University of Lyon in 1877, and named director of the newly-constructed observatory in 1879 [Véron, Véron *et al.* 2016].

In Lyon, André created a section for the study of variable stars in 1898, and charged Michel Luizet with its operation [Luizet 1912, 8]. He also lectured on recent research on variables, and wrote a two-volume textbook on the subject, published in 1899 and 1900. In the preface to the first volume, André wrote that his "main objective" in publishing his lectures, was to

[...] contribute to a return to favor, in our country, of the branch of observational astronomy [...] that, for various reasons, is a bit neglected at present [...].²⁴

The idea that astrophysics was neglected in France was shared by many, including Henri Poincaré.²⁵

Before discussing Poincaré's view of French astrophysics, a few words about his bibliography are in order. Poincaré published over 700 titles in his lifetime,

stärkere Einkerbung und vermutlich zuletzt eine Spaltung der Masse in zwei ungleiche Teile erfolgen wird. [...] Herr Poincaré hält es für zu gewagt, aus dieser Geschichte einer stets homogen bleibenden Masse auf die Umgestaltung des von vornherein inhomogenen Laplace'schen Nebels schliessen zu wollen. Denkt man aber nicht an eine Gasmasse, sondern an eine Flüssigkeit, die auch bei verschwindendem Druck an der Oberfläche stets eine endliche Dichte behält, wie sie die Erde zu einer gewissen Epoche gewesen sein mag, so darf man folgern, dass auch eine solche Flüssigkeit bei zunehmender Kontraktion einmal die Rotationsform verlieren und sich schliesslich spalten wird" [Schwarzschild 1898, 296].

24. "[...] mon but principal est de contribuer à remettre en faveur, dans notre Pays, cette branche de l'Astronomie d'observation [...] qui, par des causes diverses, y est actuellement un peu délaissée [...]" [André 1899, V].

25. On the social history of astrophysics in France, see the theses by Le Gars [2007] and Saint-Martin [2008].

including several prefaces, and reviews of research papers, but only two book reviews. His review of the first volume of André's *Traité* was his first, and his review of André's second volume was the last he would ever write. One wonders what drew him to this particular work?

Perhaps Poincaré took on the review, published in the journal he directed, because like André, he felt that French astrophysics was not keeping pace with international developments, and that André's treatise could help fortify this emergent discipline in France. Poincaré agreed with André's estimation that astrophysics was neglected in France, and applauded him not only for the "very great service to students" that the volume represented, but also for "attracting attention to these question so mysterious, so grandiose and so endearing" [Poincaré 1899, 124].²⁶

There was certainly another reason for Poincaré's review, as it was the occasion for him to express his views on current trends in astrophysics, and in particular, on statistical astronomy. The possibility envisaged by André of determining the distribution of the few stars with known parallax remained for Poincaré an excessively speculative venture, because "one is always reduced to the adventurous statistical data".²⁷

The book review was also the occasion for Poincaré to correct an error. A variable star, André reasoned, whose light curve shows a periodicity that is itself variable, allows us to discern its radial velocity, provided that the variation is on the order of a ten-thousandth part of its period. Poincaré was much concerned at this point of his career with the principle of relative motion, with respect to the electron theories of Joseph Larmor and H.-A. Lorentz, and Hertz's electrodynamics of moving media.²⁸ He noticed that, contrary to André's reasoning, under uniform motion, the dynamics of an eclipsing binary can not change. In order for the period to change, Poincaré pointed out, the variable star would have to undergo radial acceleration. This is true, as far as it goes, but neither Poincaré nor André entertained here the possibility that variables are not in a state of equilibrium, although André went on to make such a suggestion in the second volume of his treatise.²⁹

The second volume of André's *Traité d'astronomie* is devoted to double stars, multiple stars and globular nebulae. André takes care to acknowledge and repair the error in the first volume pointed out by Poincaré. Among the double stars André takes up in this volume, β Lyrae is the most prominent, as André provides a detailed résumé of Myers' model, and he agrees fully with the interpretation stressed by Myers in Williams Bay:

26. Maurice Hamy, an astronomer at the Paris Observatory, also reviewed André's lectures, which he described as "first-rate". He agreed with André's assessment that his topic was "ignored almost completely" in France [Hamy 1901].

27. "[...] on est toujours réduit aux données aventureuses de la Statistique" Poincaré [1899, 126].

28. Poincaré's view of the principle of relative motion and the principle of reaction in 1900 is explained in detail by Darrigol [2023].

29. Both β Lyrae and U Pegasi were later considered to exhibit mass transfer.

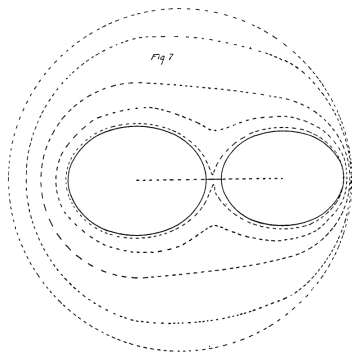


Figure 6: Equipotential surfaces of Myers' model of β Lyrae with two orbiting ellipsoids, featuring an apiodal surface [Myers 1898a].

The binary system β Lyrae constitutes [...] an absolutely remarkable case: [...] its two components are nearly in contact with each other. [...] The system U Pegasi is probably quite analogous, but with a closer contact of the two components. With such low mean densities, these two systems are probably still in a nebulous state; their atmospheres are indistinct and, intermixing at a certain distance from the stellar core, are distributed in equipotential surfaces differing more and more from a spherical form and taking on the forms so well studied by the eminent geometer Poincaré.³⁰

The equipotential surfaces were drawn by Myers in his Williams Bay paper; André reproduced them in turn for his readers (Figure 6).

Poincaré appears to have been pleased with the second volume of André's treatise, which he referred to in his presidential address to the Astronomical Society of France as an "excellent book" [Poincaré 1902b]. Similarly, in his review for the *Bulletin astronomique*, Poincaré observed that this new volume "cedes nothing to the first" with respect to the interest of its contents [Poincaré 1901].

Here again, however, Poincaré corrected an error that involved, once again, the principle of relative motion. In the case of a close binary, André

30. "Le système binaire de β Lyre constitue [...] un cas absolument remarquable: [...] ses deux composantes sont presque en contact l'une avec l'autre. Il est d'ailleurs probable que le système de U Pégase lui est fort analogue, mais avec un contact plus assuré des deux composantes. Avec d'aussi faibles densités moyennes, ces deux systèmes sont probablement encore à l'état nébuleux; leurs atmosphères se confondent et, se mêlant à une certaine distance des noyaux stellaires, se distribuent en surfaces équipotentielle différant de plus en plus de la forme sphérique et affectant les formes si bien étudiées par l'éminent géomètre Poincaré" [André 1900, 303].

explained, when the companion eclipses the principal star, its radial velocity vanishes, such that the observed radial velocity of the principal fixes the velocity of the companion with respect to the principal star [André 1900, 79]. Poincaré pointed out the obvious fact that at the moment of conjunction the companion has null radial velocity with respect to the principal. Consequently, observation of spectral lines of the principal star at conjunction fixes only the relative motion of the system's center of gravity with respect to the Earth.

This was the only fault Poincaré found with André's second volume, which he went on to present in detail. He reserved his most lyrical prose for the section on close binaries, the method of observation of which he singled out for praise. In fact, what he admired was just André's dutiful summary of Myers' thesis. This part of Poincaré's review merits a lengthy citation:

We are pleased to see united and related all these recent discoveries, which over that last few years seem to give a glimpse into a whole new world. These new views, so endearing and seductive, remain in part hypothetical. There are, nonetheless, conclusions which at present appear to have acquired a high degree of probability. The variability of Algol-type stars appears to be due to eclipses, and that of β Lyrae-type stars, to the considerable flattening of the two components which, during their rotation and orbital revolution, would present to the observer's eye at one moment a small section, and at another moment a large one. The former would be completed binary systems, the others, binary systems being formed.³¹

The view expressed here by Poincaré, that classes of variables are eclipsing binaries or proto-binaries, was shared by many theorists at the time. Poincaré was likely aware of Vogel's admission, in his 1900 review of a decade of progress in stellar motion determination, that Myers' orbit "very satisfactorily represented" the light curve of β Lyrae [Vogel 1900].³²

Poincaré's public embrace of Myers' interpretation of the pear in his glowing review of André's textbook undoubtedly attracted notice among astronomers and astrophysicists. It may well have come to the attention of Myers, who wrote to Poincaré in September, 1901, some eight months after his review appeared in the *Bulletin*. In his letter, which is transcribed in the appendix, Myers mentioned André's résumé of his research on β Lyrae and U Pegasi, and

31. "On aime à voir réunies et rapprochées toutes ces découvertes récentes qui depuis quelques années semblent nous ouvrir un aperçu sur un monde tout nouveau. Ces vues nouvelles, si attachantes et si séduisantes, sont encore en partie hypothétiques. La variabilité des étoiles du type d'Algol paraît due à des éclipses, celle des étoiles du type β de la Lyre à l'aplatissement considérable des deux composantes qui, pendant leur rotation et leur révolution orbitale, présenteraient à l'œil de l'observateur tantôt une section faible, tantôt une section considérable. Les premières seraient des systèmes binaires formés, les autres des systèmes binaires en voie de formation" [Poincaré 1901, 44].

32. Similarly, Newcomb [1902, 112] adopted Myers' view of β Lyrae and U Pegasi as eclipsing binaries, and added a third example: ζ Herculis.

presented his plan to pursue this line of research, since “by computing light curves based upon any of your other theoretical forms—perhaps the apiodal” he might reproduce the light curves of more variables. To further this project, Myers requested a copy of Poincaré’s publications on equilibrium forms of rotating fluid masses. This would help him, Myers explained to Poincaré,

to accomplish something which may be of interest to you as furnishing tangible proof of the existence in the universe of such equilibrium forms as your matchless pen has proved possible. (Annex, p. 180)

Myers’ request suggests that he was not yet fully acquainted with Poincaré’s scientific publications on equilibrium figures. This is quite plausible, as his calculation of the light curve of β Lyrae required only equilibrium figures of revolution, and not the figures discovered by Jacobi or Poincaré.

We don’t know if Poincaré responded to Myers’ letter, or if he sent the requested offprints. What we *do* know, is that Myers published no further on spectroscopic binaries, or on any other matter of scientific research. From 1901 until his retirement in 1929, Myers was professor of mathematics and astronomy in the College of Education at the University of Chicago [Marquis-Who’s Who 1943, 885], a professional circumstance which would have limited his access to research support and graduate students in astrophysics.

As for Poincaré, he was not done with the pear or his fission theory. His lectures at the Sorbonne for the academic year 1900-1901 were devoted to equilibrium figures of a rotating fluid mass, and were written up for publication by Léon Dreyfus [Poincaré 1902a]. Beginning in May, 1901, Darwin entertained correspondence with Poincaré on the topic of exchange of stabilities, which lasted a full year, and led to several related publications by both men.³³

5 Conclusion

When Poincaré took up the problem of determining the equilibrium figures of a rotating fluid mass in the mid-1880s, the possibility of observing a concrete realization of such figures in the solar system or the stellar universe was distant, but perhaps not beyond imagination. Spectroscopic binaries did not exist when Poincaré published his first work on equilibrium figures, but within four years, they did, and within ten years, Myers’ model of β Lyrae led him—and many other theorists—to believe that it was not a true binary star, but a proto-binary in the form of a Poincaré pear. Myers’ fellow doctoral student in Munich, and a rising star of German astrophysics, Karl Schwarzschild was in full agreement, as he publicly chided Poincaré for his initial reluctance

33. See the annotated transcriptions of Darwin’s and Poincaré’s letters in [Walter, Nabonnand *et al.* 2016], and the introduction by Ralf Krömer to this fascinating exchange.

to abandon the nebular hypothesis. In France, soon after the discovery of spectroscopic binaries, Poincaré brought his fission theory to the attention of a broad readership, while Charles André wrote a textbook on Myers' genial method of reducing light curves to orbital elements.

We have now seen what Poincaré's precise role was in the first fifteen years or so of the "intoxicating" history of the pear in cosmogony. No one would argue that either the Poincaré-Darwin fission theory, or the pear itself stood as a necessary condition for Myers' model of β Lyrae. It also seems likely that André's presentation of Myers' method of reducing β Lyrae's light curve to orbital elements was not wholly premised on Myers' computation of the system's equipotential surfaces. Chandrasekhar, I believe, was right about Poincaré's considerable power of persuasion. His somewhat negative characterization of Poincaré's role in the history of the pear, however, has obscured the importance of the Poincaré pear and of Poincaré-Darwin fission theory to astrophysical interpretation of a class of variable stars as eclipsing binaries or proto-binaries, the prototype of which is the pear-shaped β Lyrae.

Annex: Letter from G. W. Myers to H. Poincaré

Sept. 24, 1901

Chicago Ills. U.S.A.— 6026 Monroe Ave.

CHICAGO University – William R. Harper, PRESIDENT

Professor of Astronomy and Mathematics – School of Education

Mr. H. Poincaré, Paris France

My dear Sir:

You have perhaps noticed in Professor André's book entitled "*Traité d'astronomie stellaire*" Vol II p. 303 that my discussions of β Lyrae and U Pegasi both seem to point to a concrete confirmation of your excellent work on rotating liquids. I am now curious to see if by computing light curves based upon any of your other theoretical forms—perhaps the apiodal—I may be able to represent the light curves of any other variables. To this end I write to inquire whether you can put into my hands a copy of the paper containing your discussion, or discussions, of your various forms of rotating liquid masses in equilibrium. If you can do so, you will greatly oblige me and help me, perhaps, to accomplish something which may be of interest to you as furnishing tangible proof of the existence in the universe of such equilibrium forms as your matchless pen has proved possible.

Most respectfully yours,

G. W. Myers³⁴

34. Myers' one-page manuscript is in the Poincaré family archives, and may be consulted in digital form on the website Henri Poincaré Papers

Acknowledgments

This paper was presented at the Poincaré meeting in Nancy organized by the Henri-Poincaré Archives in July 2022. I thank the meeting participants, an anonymous referee, and the editors for comments. I am also grateful to the Poincaré Estate for sharing Henri Poincaré's correspondence and authorizing its publication.

Bibliography

- AITKEN, Robert Grant [1918], *The Binary Stars*, New York: Douglas C. McMurtrie, URL <https://archive.org/details/binarystars00aitk>.
- ANDRÉ, Charles [1899], *Traité d'astronomie stellaire*, vol. 1: Étoiles simples, Paris: Gauthier-Villars, URL <https://archive.org/details/traitdastronomi01andrgoog>.
- [1900], *Traité d'astronomie stellaire*, vol. 2: Étoiles doubles et multiples, amas stellaires, Paris: Gauthier-Villars, URL <https://archive.org/details/traitdastronomi00andrgoog>.
- ARCHIBALD, Thomas [2015], Saturn's rings from Laplace to Poincaré, in: *A Delicate Balance: Global Perspectives on Innovation and Tradition in the History of Mathematics, A Festschrift in Honor of Joseph W. Dauben*, edited by David E. Rowe & Wann-Sheng Horng, Basel: Birkhäuser, 103–124.
- BEER, Gillian [1989], 'The Death of the Sun': Victorian solar physics and solar myth, in: *The Sun is God: Painting, Literature, and Mythology in the Nineteenth Century*, edited by J. B. Bullen, Oxford: Oxford University Press, 159–180.
- BELOPOLSKY, Aristarkh [1894], Les changements dans le spectre de β Lyrae, *Memorie della Societa degli Spettroscopisti Italiani*, 22, 101–111, URL <https://articles.adsabs.harvard.edu/pdf/1894MmSSI..22..101B>.
- BRUSH, Stephen G. [1996], *A History of Modern Planetary Physics*, vol. 1: Nebulous Earth, The Origin of the Solar System and the Core of the Earth from Laplace to Jeffreys, Cambridge: Cambridge University Press, URL <https://archive.org/details/historyofmodernp0000brus>.
- CAMPBELL, William Wallace [1913], *Stellar Motions, With Special Reference to Motions Determined by Means of the Spectrograph*, New Haven: Yale University Press, URL <https://archive.org/details/stellarmotionswi00camp>.

- CASE, Stephen [2018], *Making Stars Physical: The Astronomy of Sir John Herschel*, Pittsburgh: University of Pittsburgh Press.
- CHANDLER JR., Seth Carlo [1888], Catalogue of variable stars, *Astronomical Journal*, 8(11–12), 81–94, doi: 10.1086/101060.
- [1893], Second catalogue of variable stars, *Astronomical Journal*, 13(12), 89–110, doi: 10.1086/101922.
- [1896], Third catalogue of variable stars, *Astronomical Journal*, 16(19), 145–172, doi: 10.1086/102484.
- CHANDRASEKHAR, Subrahmanyan [1969], *Ellipsoidal Figures of Equilibrium*, New Haven: Yale University Press.
- DARRIGOL, Olivier [2023], Poincaré and the reaction principle in electrodynamics, *Philosophia Scientiæ*, 27(2), 63–125.
- DARWIN, George Howard [1880], On the secular changes in the elements of the orbit of a satellite revolving about a tidally distorted planet, *Philosophical Transactions of the Royal Society of London*, 171, 713–891, doi: 10.1098/rspl.1879.0076.
- [1887], On figures of equilibrium of rotating masses of fluid, *Philosophical Transactions of the Royal Society of London*, 178, 379–428, doi: 10.1098/rsta.1887.0013.
- [1909], The genesis of double stars, in: *Darwin and Modern Science*, edited by A. C. Seward, Cambridge: Cambridge University Press, 543–564, URL <https://archive.org/details/darwinandmoderns028656mbp>.
- DEWEY, John [1902], The University of Chicago School of Education, *Elementary School Teacher*, 3(3), 200–203.
- DJURAŠEVIĆ, G., ROVITHIS-LINANIOU, H., *et al.* [2001], A photometric study of the W UMa-type system U Pegasi, *Astronomy & Astrophysics*, 367(3), 840–847, doi: 10.1051/0004-6361:20000493.
- GRAY, Jeremy J. [2013], *Henri Poincaré: A Scientific Biography*, Princeton: Princeton University Press.
- GREENBERG, John Leonard [1995], *The Problem of the Earth's Shape from Newton to Clairaut: The Rise of Mechanical Science in Eighteenth-Century Paris, and the Fall of "Normal" Science*, Cambridge: Cambridge University Press, URL <https://archive.org/details/problemofearthss1995gree>.
- GUICCIARDINI, Niccolò [1989], *The Development of Newtonian Calculus in Britain, 1700–1800*, Cambridge: Cambridge University Press.

- GYLDÉN, Hugo [1880], Versuch einer mathematischen Theorie zur Erklärung des Lichtwechsels der veränderlichen Sterne, *Acta Societatis scientiarum Fennicae*, 11, 345–407, URL <https://hdl.handle.net/2027/umn.31951d006952672>.
- HALE, George Ellery [1898], The Harvard conference, *Astrophysical Journal*, 8, 193–198, doi: 10.1086/140518.
- HAMY, Maurice [1901], André (Ch.), *Traité d'Astronomie stellaire*, II^e partie: Étoiles doubles et multiples, Amas stellaires, *Revue générale des sciences pures et appliquées*, 12, 931–932, URL <https://gallica.bnf.fr/ark:/12148/bpt6k948109>.
- HARTING, Carl August Johannes Hans [1889], *Untersuchungen über den Lichtwechsel des Sternes β Persei*, Ph.D. thesis, Ludwig-Maximilians University.
- HEARNSHAW, John B. [1996], *The Measurement of Starlight: Two Centuries of Astronomical Photometry*, Cambridge: Cambridge University Press, URL <https://archive.org/details/measurementofsta0000hear>.
- [2014], *The Analysis of Starlight: Two Centuries of Astronomical Spectroscopy*, Cambridge: Cambridge University Press.
- HENROTEAU, François Charles [1928], Double and multiple stars, in: *Handbuch der Astrophysik*, edited by F. C. Henroteau, H. Ludendorff, K. G. Malmquist, & F. J. M. Stratton, Berlin: Springer, vol. 6: Das Sternsystem, Volume 2, 299–470.
- HEPPERGER, Joseph von [1910], Bahnbestimmung der Doppelsterne und Satelliten, in: *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen VI, Part 2: Astronomie*, edited by Karl Schwarzschild & Rudolf Lehmann-Filhés, Leipzig: Teubner, 463–511, URL <http://resolver.sub.uni-goettingen.de/purl?PPN360833608>.
- HERRMANN, Dieter B. [1984], *The History of Astronomy from Herschel to Hertzprung*, Cambridge: Cambridge University Press, URL <https://archive.org/details/lish00diet>.
- HERSCHEL, John F. W. [1850], On the determination of the most probable orbit of a binary star, *Memoirs of the Royal Astronomical Society*, 18, 47–68, URL <https://ui.adsabs.harvard.edu/abs/1850MmRAS..18...47H>.
- HOGG, Helen Sawyer [1984], Variable stars, in: *The General History of Astronomy*, edited by Owen Gingerich, Cambridge: Cambridge University Press, vol. 4: Astrophysics and Twentieth-Century Astronomy to 1950, Part 1, 73–89, URL https://archive.org/details/generalhistoryof0000unse_d4p5.

- HOSKIN, Michael [1979], Goodricke, Pigott and the quest for variable stars, *Journal for the History of Astronomy*, 10, 23–41, doi: 10.1177/002182867901000103.
- JARDETZKY, Wenceslas S. [1958], *Theories of Figures of Celestial Bodies*, New York: Interscience, URL <https://archive.org/details/dli.ernet.176930>.
- JEANS, James Hopwood [1903], On the equilibrium of rotating liquid cylinders, *Philosophical Transactions of the Royal Society A*, 200, 67–104, doi: 10.1098/rsta.1903.0002.
- KUIPER, Gerard P. [1941], On the interpretation of β Lyrae and other close binaries, *Astrophysical Journal*, 93, 133–177, doi: doi.org/10.1086/144252.
- LE GARS, Stéphane [2007], *L'Émergence de l'astronomie physique en France (1860–1914) : acteurs et pratiques*, Ph.D. thesis, Université de Nantes, URL http://hal.archives-ouvertes.fr/docs/00/40/50/48/PDF/These_Stephane_Le_Gars.pdf.
- LOCKYER, J. Norman [1890], *The Meteoritic Hypothesis: A Statement of the Results of a Spectroscopic Inquiry into the Origin of Cosmical Systems*, London: Macmillan, URL <https://archive.org/details/meteoritichypoth00lockrich>.
- LUIZET, Michel [1912], *Les Céphéides considérées comme étoiles doubles, avec une monographie de l'étoile variable δ Céphée*, Ph.D. thesis, Université de Lyon, URL <https://hdl.handle.net/2027/mdp.39015073350368>.
- LÜTZEN, Jesper [1984], Joseph Liouville's work on the figures of equilibrium of a rotating mass of fluid, *Archive for History of Exact Sciences*, 30, 113–166, doi: 10.1007/BF00330238.
- LYTTLETON, Raymond Arthur [1953], *The Stability of Rotating Liquid Masses*, Cambridge: Cambridge University Press, URL <https://archive.org/details/stabilityofrotat032172mbp>.
- MARQUIS-WHO'S WHO (ed.) [1943], *Who Was Who in America*, vol. 1: 1897–1942, Chicago: Marquis-Who's Who, URL <https://archive.org/details/whowaswhoinameri01peri>.
- MAWHIN, Jean [2014], The implicit function theorem and its substitutes in Poincaré's qualitative theory of differential equations, *Studies in History and Philosophy of Modern Physics*, 47(1), 124–130, doi: 10.1016/j.shpsb.2014.01.006.
- MEADOWS, Arthur Jack [2008], *Science and Controversy: A Biography of Sir Norman Lockyer*, London: Macmillan.

- MÜLLER, Gustav & HARTWIG, Ernst (eds.) [1918], *Geschichte und Literatur des Lichtwechsels, der bis Ende 1915 als sicher veränderlich anerkannten Sterne: nebst einem Katalog der Elemente ihres Lichtwechsels*, vol. 1, Leipzig: Poeschel & Trepte, URL <https://hdl.handle.net/2027/pst.000004069942>.
- MYERS, George William [1896], *Untersuchungen über den Lichtwechsel des Sternes β Lyrae*, Ph.D. thesis, Ludwig-Maximilians-Universität zu München, URL <https://hdl.handle.net/2027/uiug.30112114019356>.
- [1898a], The system of β Lyrae, *Astrophysical Journal*, 7(1), 1–22, doi: 10.1086/140433.
- [1898b], The variable star U Pegasi, *Astrophysical Journal*, 8, 163–173, doi: 10.1086/140514.
- NEWCOMB, Simon [1902], *The Stars: A Study of the Universe*, New York: G.P. Putnam's Sons, URL <https://archive.org/details/starsstudyofuniv00newcrich>.
- OSTERBROCK, Donald E. [1997], *Yerkes Observatory, 1892-1950: The Birth, Near Death, and Resurrection of a Scientific Research Institution*, Chicago: University of Chicago Press.
- [1999], AAS meetings before there was an AAS: the pre-history of the Society, in: *The American Astronomical Society's First Century*, edited by David H. DeVorkin, College Park: American Institute of Physics, 3–19, URL <https://archive.org/details/americanastronom0000unse>.
- PANNEKOEK, Anton [1961], *A History of Astronomy*, London: George Allan and Unwin.
- PARSHALL, Karen Hunger & ROWE, David E. [1994], *The Emergence of the American Mathematical Research Community, 1876–1900: J. J. Sylvester, Felix Klein, and E. H. Moore*, Providence, RI; London: American Mathematical Society and London Mathematical Society, URL <https://archive.org/details/emergenceofameri0008pars>.
- PICKERING, Edward C. [1880], Dimensions of the fixed stars, with especial reference to binaries and variables of the Algol Type, *Proceedings of the American Academy of Science*, 16, 1–37, URL <https://ui.adsabs.harvard.edu/abs/1880HarPa...1....9P>.
- [1891], Spectrum of β Lyrae, *Astronomische Nachrichten*, 128(3), 39–42, doi: 10.1002/asna.18911280304.
- [1898], The variable star U Pegasi, *Harvard College Observatory Circular*, n° 23, URL <https://ui.adsabs.harvard.edu/abs/1898HarCi..23....1W>.

- POINCARÉ, Henri [1885], Sur l'équilibre d'une masse fluide animée d'un mouvement de rotation, *Acta mathematica*, 7(1), 259–380, URL <http://henripoincarepapers.univ-nantes.fr/chp/hp-pdf/hp1885ama.pdf>.
- [1892], Les formes d'équilibre d'une masse fluide en rotation, *Revue générale des sciences pures et appliquées*, 3(23), 809–815, URL <http://henripoincarepapers.univ-nantes.fr/chp/hp-pdf/hp1892rga.pdf>.
- [1899], Ch. André, *Traité d'astronomie stellaire*, *Bulletin astronomique*, 16, 124–127, URL <https://ui.adsabs.harvard.edu/abs/1899BuAsI..16..124>.
- [1901], André, *Traité d'astronomie stellaire*, 2^e partie, *Bulletin astronomique*, 18(1), 42–45, URL https://www.persee.fr/doc/bastr_0572-7405_1901_num_18_1_11658_t1_0042_0000_2.
- [1902a], *Figures d'équilibre d'une masse fluide*, Paris: C. Naud, URL <http://www.archive.org/details/figuresdquilibr00poingooq>.
- [1902b], Les progrès de l'astronomie en 1901, *Bulletin de la Société astronomique de France*, 16, 214–223, URL <http://henripoincarepapers.univ-nantes.fr/chp/hp-pdf/hp1902sa.pdf>.
- [1911], Les hypothèses cosmogoniques, *Revue du mois*, 12, 385–403, URL <http://henripoincarepapers.univ-nantes.fr/chp/text/hp1911rm.html>.
- RAMBAUT, Arthur A. [1891], On the determination of double star orbits from spectroscopic observations of the velocity in the line of sight, *Monthly Notices of the Royal Astronomical Society*, 51, 316, doi: 10.1093/mnras/51.5.316.
- ROQUE, Tatiana [2011], Stability of trajectories from Poincaré to Birkhoff: approaching a qualitative definition, *Archive for History of Exact Sciences*, 65(3), 295–342, doi: 10.1007/s00407-011-0079-0.
- SAINT-MARTIN, Arnaud [2008], *L'Office et le télescope : Une sociologie de l'astronomie française, 1900–1940*, Ph.D. thesis, Université Paris 4.
- SCHAFER, Simon [1989], The nebular hypothesis and the science of progress, in: *History, Humanity and Evolution: Essays for John C. Greene*, edited by James Richard Moore, Cambridge: Cambridge University Press, 131–164.
- SCHWARZSCHILD, Karl [1898], Die Poincaré'sche Theorie des Gleichgewichts einer homogenen rotierenden Flüssigkeitsmasse, *Neue Annalen der königlichen Sternwarte in Bogenhausen bei München*, 3, 231–299, URL <http://adsabs.harvard.edu/abs/1898AnBog...3..231S>.
- STEBBINS, Joel [1911], The discovery of eclipsing variable stars, *Astrophysical Journal*, 34, 105–111, doi: 10.1086/141874.

- STEIN, Johan [1924], *Die veränderlichen Sterne*, vol. 2: Mathematischer physikalischer Teil, Freiburg im Breisgau: Herder, URL <https://hdl.handle.net/2027/uiug.30112059289766>.
- STRUVE, Otto [1958], The problem of β Lyrae, *Proceedings of the Astronomical Society of the Pacific*, 70(412), 5–40, doi: 10.1086/127169.
- VÉRON, Philippe, VÉRON, Mira, *et al.* [2016], *Dictionnaire des astronomes français (1850–1950)*, Saint-Michel-l’Observatoire: Unpublished typescript, URL <http://www.obs-hp.fr/dictionnaire/>.
- VOGEL, Hermann Carl [1890], Spectrographische Beobachtungen an Algol, *Astronomische Nachrichten*, 123(19), 289–292, doi: 10.1002/asna.18901231902.
- [1894a], On the spectrum of β Lyrae (I), *Astronomy and Astro-Physics*, 13(5), 358–367, URL <https://hdl.handle.net/2027/uc1.b4142443>.
- [1894b], On the spectrum of β Lyrae (II), *Astronomy and Astro-Physics*, 13(8), 561–568, URL <https://hdl.handle.net/2027/uc1.b4142443>.
- [1894c], Über das Spectrum von β Lyrae, *Sitzungsberichte der königlich preußischen Akademie der Wissenschaften*, 115–132, URL <https://hdl.handle.net/2027/coo.31924087721977>.
- [1900], On the progress made in the last decade in the determination of stellar motions in the line of sight, *Astrophysical Journal*, 11, 373–392, doi: 10.1086/140707.
- WALTER, Scott A., NABONNAND, Philippe, *et al.* (eds.) [2016], *La Correspondance entre Henri Poincaré, les astronomes, et les géodésiens*, Basel: Birkhäuser.
- WILKENS, Alexander [1927], *Hugo von Seeligers wissenschaftliches Werk*, Munich: Oldenbourg, doi: 10.1515/9783486756234.
- WILLIAMS, Mari [1984], Beyond the planets: early nineteenth-century studies of double stars, *British Journal for the History of Science*, 17(3), 295–309, doi: 10.1017/S0007087400021312.
- ZÖLLNER, Friedrich [1861], *Grundzüge einer allgemeinen Photometrie des Himmels*, Berlin: Mitscher & Röstel, URL <https://hdl.handle.net/2027/hvd.32044078774080>.

