It's only a model: spacetime geometry in the transition from Galilean to relativistic kinematics

Scott Walter

[Abstract of a talk delivered on 28 May 2008 at the MFO Oberwolfach workshop on History of mathematics in the early 20th century: the role of transition, organized by Leo Corry, Della D. Fenster, and Joachim Schwermer. First published in *Oberwolfach Reports* 5(2), 2008.]

Nineteenth-century mathematical physicists were skilled in the art of model-building, particularly when it came to the luminiferous ether. Theorists of the elastic-solid ether, the hydrodynamic ether and the vortex ether proposed model after model, none of which, however, acquired the ring of truth. Some began to deplore a reliance on images altogether, in favor of a more abstract approach. The 1890s saw a development of this tendency, as Heinrich Hertz, Henri Poincaré and others in France and Germany explored alternative foundations of mechanics. At the same time, electrodynamicists set about dematerializing the ether, and confounding it with absolute space.

Poincaré, for example, reinterpreted Hertz's theory of the electrodynamics of moving bodies [5], assuming dilute matter to exist even in a perfect vacuum. This gave the Hertz force something to act on, thereby saving Newton's third law, not to mention the first law of thermodynamics, while preserving the principle of relative motion. In essence, as Darrigol [3, 356] remarked, Poincaré did away with the ether in Hertz's theory. Hertz's theory remained problematic, however, because of its incompatibility with the results of Fizeau's experiment, which indicated only partial, and not total ether drag by running water.

A few years later, in 1905, Einstein did away with the ether altogether, by introducing new kinematic assumptions implying time dilation and length contraction. Einstein [4] noted a peculiar consequence of his assumptions: the time measured by an ideal clock moving with constant speed is not absolute, but depends on the path. He did not attempt to illustrate his kinematics diagrammatically, but demonstrated that it led, via a long, convoluted calculation, to the Lorentz transformation.

Also in 1905, Poincaré noted that the Lorentz transformation forms a group, and may be represented geometrically as a coordinate rotation about the origin of a four-dimensional vector space, where the space axes are real, and the temporal axis is imaginary. In his 1906–1907 Sorbonne lectures, he presented his ideas on the principle of relativity, and to illustrate the transitivity of measurement, he employed a novel graphic device, known today as a light-ellipsoid [2, 38].

Poincaré considered the wave produced by a flash of light from a source in constant rectilinear motion with respect to an observer at rest. At some time t after the flash, a co-moving observer ascertains the radius of the light wave with a measuring rod deformed by Lorentz-FitzGerald contraction. Taking the contraction into account, the co-moving observer concludes that the rectified form of the wave is an ellipsoid. Although Poincaré did not show this, the Lorentz transformation may be derived from the geometric relations of his light ellipse (Fig. 1).

Correcting for motion in the moving frame implies knowledge of the frame velocity, and is quite contrary to an approach where all inertial frames are equivalent, such as that of Einstein. Poincaré, however, considered that there was only one frame in which measurements required no correction: the ether frame. In all other frames, measured quantities were “apparent”, not “true”.

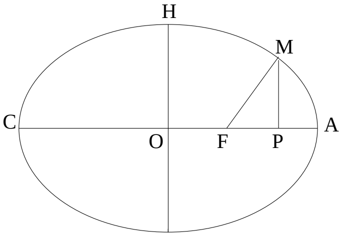

Fig. 1. Poincaré’s light ellipse

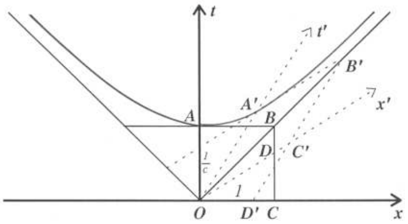

Fig. 2. Minkowski’s spacetime diagram [6]

Shortly after Poincaré published his light ellipse, Einstein’s former teacher Hermann Minkowski unveiled his theory of electrodynamics of moving media, along with a spacetime mechanics, all expressed in a novel four-dimensional vector formalism. Within two weeks, Einstein and his collaborator Jacob Laub targeted two aspects of Minkowski’s work: (1) the novel four-dimensional formalism, and (2) the new electrodynamics, which they felt to be inconsistent with the observational base. Any reviewer of Minkowski’s electrodynamics had to either rewrite his equations in a recognizable form, or provide a precis of his formalism. Einstein and Laub chose the former route, along with Max Abraham, and Gunnar Nordström. With hindsight, it is clear that Minkowski committed a serious tactical error by coupling his formalism to a controversial electrodynamics of moving media.

Minkowski probably came to this conclusion himself, as he devoted his next publication, entitled “Raum und Zeit,” almost entirely to elements of his spacetime geometry and mechanics, with little consideration of electrodynamics.[1] His lecture introduced a crucial tool for research in relativity theory: the spacetime diagram (Fig. 2). While displacement diagrams were a common sight in contemporary textbooks on mechanics, the only comparable illustrative technique available in relativity theory was Poincaré’s unwieldy light ellipse.

The spacetime diagram is a central feature of Minkowski’s Cologne lecture, elegantly illustrating the limit relation between pre-relativist and relativist mechanics. Minkowski observed that as the value of c approaches infinity, the primed space and time axes, x' and t', of the moving frame on a spacetime diagram (Fig. 2) collapse symmetrically into the unprimed space and time axes, x and t,

---

[1] In “Raum und Zeit,” Minkowski provided a new geometric view (via a spacetime diagram) of the Liénard-Wiechert potential in terms of a four-potential, and expressed the four-force between two electrons in arbitrary motion.

of pre-relativist mechanics.[2] Minkowski also employed the spacetime diagram in a misguided attempt to distinguish his spacetime theory from Einstein's theory of relativity, an error Born [1, 246] and other writers sought to correct [7, § 2.4].

The evolution of the content of these three publications, when taken in sequence, is quite striking. First, Minkowski proposes an electrodynamics of moving media, expressed in a novel four-dimensional calculus. Next, Einstein and Laub excise the four-dimensional formalism from Minkowski's electrodynamics of moving media, rewriting the latter in the usual three-dimensional form. Finally, Minkowski suppresses electrodynamics from his theory of spacetime. As a result of Minkowski's censorship, "Raum und Zeit" successfully focused attention on the transformation group leaving invariant the laws of physics, much as Poincaré and Einstein had tried in vain to do three years earlier.

Minkowski's spacetime diagram provided a comprehensible graphic illustration of the kinematics of the Lorentz group. With practice, the unintuitive effects of relativity (Lorentz-FitzGerald contraction, time dilation) were understood as direct consequences of geometric relations inscribed in the spacetime diagram. Although Minkowski did not show this himself, when illustrated on a Minkowski diagram, Poincaré's light ellipsoid corresponds precisely to the projection of a light sphere contained in a certain constant-time hyperplane of a moving frame on the spacelike hyperplane $t = 0$ of a frame at rest.

After further contributions by Sommerfeld and Laue, physicists – Einstein included – recognized the advantages of a four-dimensional approach to relativity, and contributed to a growing corpus of Minkowskian relativity. From the standpoint adopted here, Minkowski's spacetime diagram appears as a crucial element in the transition from classical to relativistic kinematics.

References

[1] Max Born. *Einstein's Theory of Relativity.* Dover, New York, 2d edition, 1962.

[2] Olivier Darrigol. Henri Poincaré's criticism of fin de siècle electrodynamics. *Studies in History and Philosophy of Modern Physics* 26 (1995), 1–44.

[3] Olivier Darrigol. *Electrodynamics from Ampère to Einstein.* Oxford University Press, Oxford, 2000.

[4] Albert Einstein. Zur Elektrodynamik bewegter Körper. *Annalen der Physik* 17 (1905), 891–921.

[5] Heinrich Hertz. Über die Grundgleichungen der Elektrodynamik für bewegte Körper. *Annalen der Physik und Chemie* 41 (1890), 369–398.

[6] Hermann Minkowski. Raum und Zeit. *Jahresbericht der deutschen Mathematiker-Vereinigung* 18 (1909), 75–88.

[7] Scott Walter. Minkowski, mathematicians, and the mathematical theory of relativity. In Hubert Goenner, Jürgen Renn, Tilman Sauer, and Jim Ritter, editors, *The Expanding Worlds of General Relativity*, volume 7 of *Einstein Studies*, pages 45–86. Birkhäuser, Boston/Basel, 1999.

---

[2] The obliquity of coordinate axes on a spacetime diagram is actually an artifact of the representation, as these axes are orthogonal in Minkowski geometry.